



Wydział Mechaniczny Energetyki i Lotnictwa
Zakład Wytrzymałości Materiałów i Konstrukcji

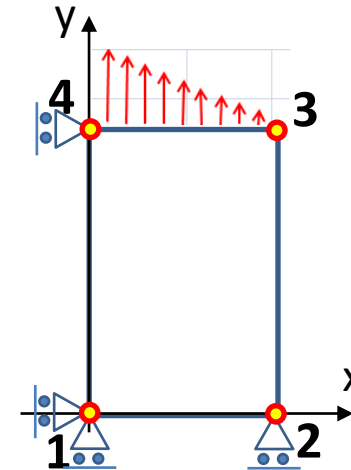
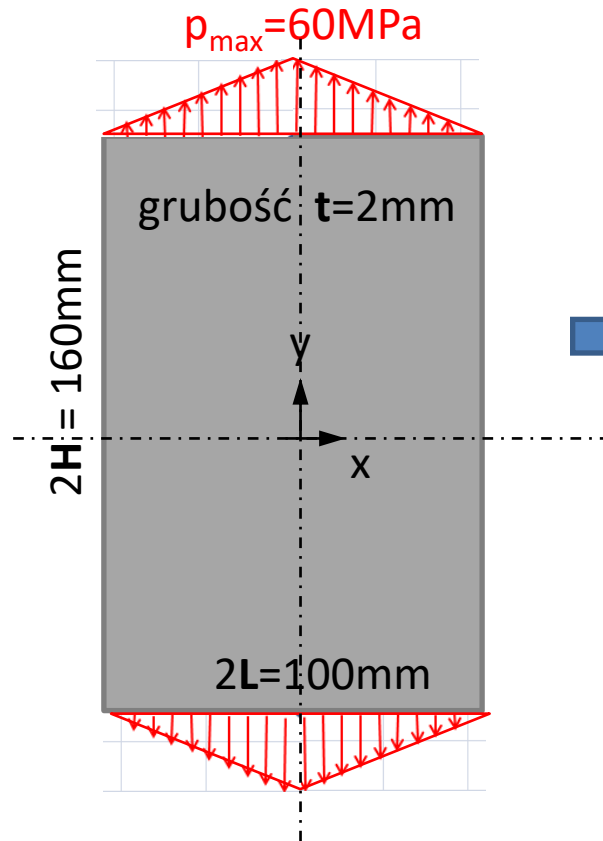


Metoda elementów skończonych (MES1)

Wykład 6C. Tarcza 2D modelowana elementami 4-węzłowymi

03.2024

Przykład. 2D tarcza model MES z użyciem elementów 4-węzłowych



Model ćwiartki tarczy
o jednym elemencie skończonym

wektory współrzędnych węzłowych:

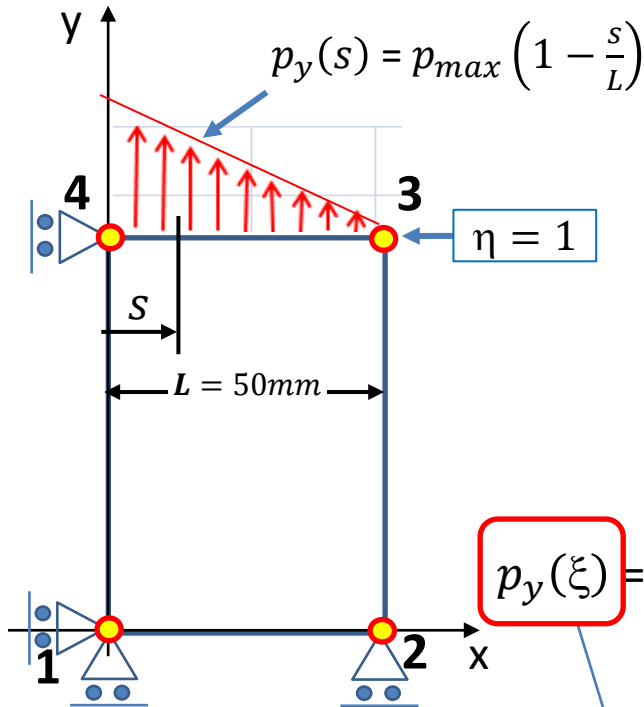
$$[x_i]_1 = [0, L, L, 0]$$

$$[y_i]_1 = [0, 0, H, H]$$

Wektor obciążenia równoważnego od sił powierzchniowych

$$[F^p]_e = t_e \int_{-1}^1 [p_x, p_y] [N] \sqrt{\left(\frac{\partial [N(\xi,1)]}{\partial \xi} \{x_i\}_e\right)^2 + \left(\frac{\partial [N(\xi,1)]}{\partial \xi} \{y_i\}_e\right)^2} d\xi$$

1×8 2×8 1×4 4×1 1×4 4×1



$$\begin{aligned} x_3 &= L, & x_4 &= 0 \\ y_3 &= H, & y_4 &= H \end{aligned}$$

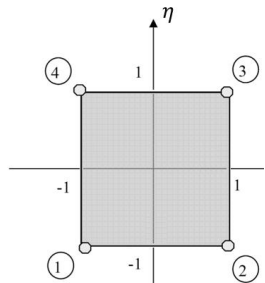
$$\left. \begin{aligned} s=0 &\rightarrow \xi = -1 \\ s=L &\rightarrow \xi = +1 \end{aligned} \right\} \rightarrow s(\xi) = \frac{L}{2}(\xi + 1)$$

$$p_y(\xi) = p_{max} \left(1 - \frac{s(\xi)}{L}\right) = p_{max} \left(1 - \frac{1}{2}(\xi + 1)\right) = \frac{p_{max}}{2} (1 - \xi)$$

$$[F^p]_1 = t \int_{-1}^1 [0, p_y(\xi)] [N] \sqrt{\left(\frac{\partial [N(\xi,1)]}{\partial \xi} \{x_i\}_e\right)^2 + \left(\frac{\partial [N(\xi,1)]}{\partial \xi} \{y_i\}_e\right)^2} d\xi$$

1×8 2×8 1×4 4×1 1×4 4×1

układ współrzędnych naturalnych

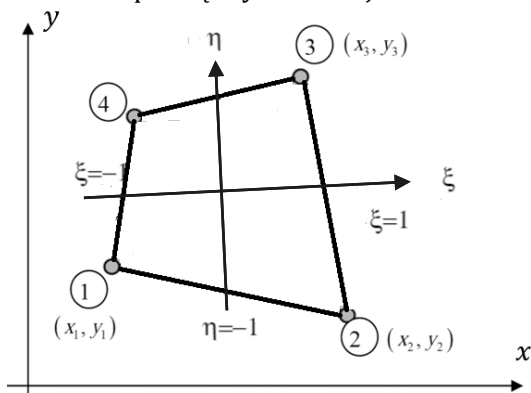


funkcje kształtu elementu i ich pochodne w układzie naturalnym:

$$\begin{aligned}
 N_1(\xi, \eta) &= \frac{1}{4}(1 - \xi)(1 - \eta) \\
 N_2(\xi, \eta) &= \frac{1}{4}(1 + \xi)(1 - \eta) \\
 N_3(\xi, \eta) &= \frac{1}{4}(1 + \xi)(1 + \eta) \\
 N_4(\xi, \eta) &= \frac{1}{4}(1 - \xi)(1 + \eta)
 \end{aligned}$$

i	$N_i(\xi, 1)$
1	0
2	0
3	$\frac{1}{2}(1 + \xi)$
4	$-\frac{1}{2}(1 - \xi)$

układ współrzędnych kartezjańskich



i	$\frac{\partial N_i}{\partial \xi}$	$\frac{\partial N_i}{\partial \eta}$
1	$-\frac{1}{4}(1 - \eta)$	$-\frac{1}{4}(1 - \xi)$
2	$\frac{1}{4}(1 - \eta)$	$-\frac{1}{4}(1 + \xi)$
3	$\frac{1}{4}(1 + \eta)$	$\frac{1}{4}(1 + \xi)$
4	$-\frac{1}{4}(1 + \eta)$	$\frac{1}{4}(1 - \xi)$



$\frac{\partial N_i(\xi, 1)}{\partial \xi}$
$-\frac{1}{4}(1 - 1) = 0$
$\frac{1}{4}(1 - 1) = 0$
$\frac{1}{4}(1 + 1) = \frac{1}{2}$
$-\frac{1}{4}(1 + 1) = -\frac{1}{2}$

$$\begin{aligned} x_3 &= L, \quad x_4 = 0 \\ y_3 &= H, \quad y_4 = H \end{aligned}$$

$$\begin{aligned} \frac{\partial N_3(\xi, 1)}{\partial \xi} &= \frac{1}{2}, \quad \frac{\partial N_4(\xi, 1)}{\partial \xi} = -\frac{1}{2} \\ N_3(\xi, 1) &= \frac{1}{2}(1 + \xi), \quad N_4(\xi, 1) = -\frac{1}{2}(1 - \xi) \end{aligned}$$

$$[F^p]_1 = t \int_{-1}^1 [0, p_y(\xi)] [N] \sqrt{\left(\frac{\partial [N(\xi, 1)]}{\partial \xi} \{x_i\}_e \right)^2 + \left(\frac{\partial [N(\xi, 1)]}{\partial \xi} \{y_i\}_e \right)^2} d\xi$$

$$\frac{\partial [N(\xi, 1)]}{\partial \xi} \{x_i\}_1 = \frac{\partial N_3(\xi, 1)}{\partial \xi} x_3 + \frac{\partial N_4(\xi, 1)}{\partial \xi} x_4 = \frac{1}{2} \cdot L - \frac{1}{2} \cdot 0 = \frac{L}{2}$$

$$\frac{\partial [N(\xi, 1)]}{\partial \xi} \{y_i\}_1 = \frac{\partial N_3(\xi, 1)}{\partial \xi} y_3 + \frac{\partial N_4(\xi, 1)}{\partial \xi} y_4 = \frac{1}{2} \cdot H - \frac{1}{2} \cdot H = 0$$

$$\sqrt{\left(\frac{\partial [N(\xi, 1)]}{\partial \xi} \{x_i\}_e \right)^2 + \left(\frac{\partial [N(\xi, 1)]}{\partial \xi} \{y_i\}_e \right)^2} = \frac{L}{2}$$

$$[F^p]_1 = t \frac{L}{2} \int_{-1}^1 [0, p_y(\xi)] [N(\xi, 1)] d\xi$$

$$[N(\xi, 1)] = \begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) \end{bmatrix}$$

$$N_3(\xi, 1) = \frac{1}{2}(1 + \xi) \quad , \quad N_4(\xi, 1) = -\frac{1}{2}(1 - \xi)$$

$$[N(\xi, 1)] = \begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) \end{bmatrix}$$

$$[F^p]_1 = t \frac{L}{2} \int_{-1}^1 [0, p_y(\xi)] [N(\xi, 1)] d\xi$$

$$p_y(\xi) = \frac{p_{max}}{2} (1 - \xi)$$

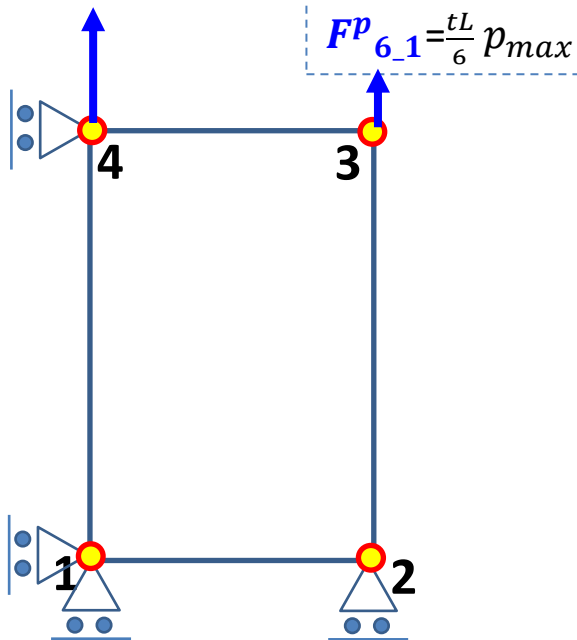
$$\begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) \end{bmatrix}$$

$$[0, p_y(\xi)] \quad [0, 0, 0, 0, 0, N_3(\xi, 1)p_y(\xi), 0, N_4(\xi, 1)p_y(\xi)]$$

$$F^p_{8_1} = \frac{tL}{3} p_{max}$$

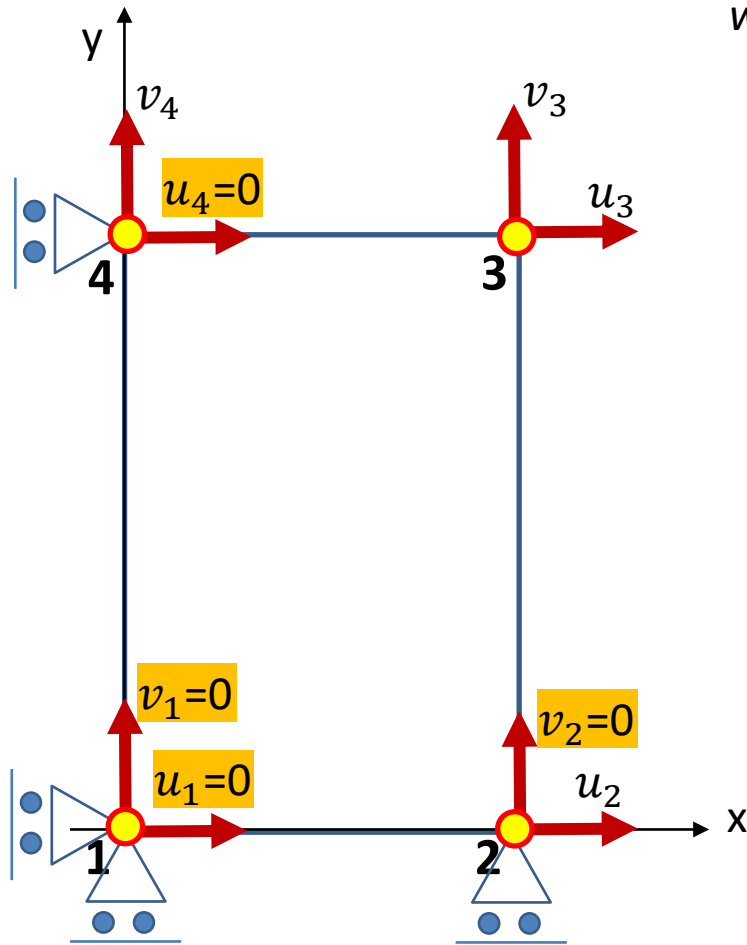
$$F^p_{6_1}$$

$$F^p_{8_1}$$



$$F^p_{6_1} = t \frac{L}{2} \int_{-1}^1 \frac{p_{max}}{2} (1 - \xi) \frac{1}{2}(1 + \xi) d\xi = \frac{tL}{6} p_{max}$$

$$F^p_{8_1} = t \frac{L}{2} \int_{-1}^1 \frac{p_{max}}{2} (1 - \xi) (-\frac{1}{2}(1 - \xi)) d\xi = \frac{tL}{3} p_{max}$$



wektor parametrów węzłowych elementu 1

$$\{q\}_1 = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}_1 \quad \text{wektor - kolumna}$$

8×1

$$[q]_1 = [u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4]_1$$

1×8

wektor - wiersz

Warunki brzegowe: $N = 8 - 4 = 4$

wektor aktywnych parametrów węzłowych po uwzględnieniu warunków brzegowych:

$$[q]_1 = [u_2, u_3, v_3, v_4]_1$$

1×4

Macierz gradientu dla warunku PSN:

$$\begin{aligned}
 [R]_{3 \times 2} &= \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \frac{\partial}{\partial \eta} \right) & 0 \\ 0 & \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \frac{\partial}{\partial \eta} - \frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \frac{\partial}{\partial \xi} \right) \\ \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \frac{\partial}{\partial \eta} - \frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \frac{\partial}{\partial \xi} \right) & \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \frac{\partial}{\partial \eta} \right) \end{bmatrix} = \\
 &= \begin{bmatrix} \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \frac{\partial}{\partial \xi} \right) & 0 \\ 0 & \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \frac{\partial}{\partial \xi} \right) \\ \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \frac{\partial}{\partial \xi} \right) & \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \frac{\partial}{\partial \xi} \right) \end{bmatrix} + \begin{bmatrix} \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \frac{\partial}{\partial \eta} \right) & 0 \\ 0 & \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \frac{\partial}{\partial \eta} \right) \\ \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \frac{\partial}{\partial \eta} \right) & \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \frac{\partial}{\partial \eta} \right) \end{bmatrix} = \\
 &= \begin{bmatrix} \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \right) & 0 \\ 0 & \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \right) \\ \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \right) & \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \right) \end{bmatrix} \frac{\partial}{\partial \xi} + \begin{bmatrix} \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \right) & 0 \\ 0 & \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \right) \\ \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \right) & \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \right) \end{bmatrix} \frac{\partial}{\partial \eta}
 \end{aligned}$$

macierz odkształcenie–przemieszczenie

$$[B]_{3 \times 16} = [R]_{3 \times 2} [N]_{2 \times 16} = \begin{bmatrix} \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \right) & 0 \\ 0 & \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \right) \\ \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \right) & \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \right) \end{bmatrix} \begin{bmatrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{bmatrix} + \begin{bmatrix} \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \right) & 0 \\ 0 & \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \right) \\ \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \right) & \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \right) \end{bmatrix} \begin{bmatrix} \frac{\partial N}{\partial \eta} \\ \frac{\partial N}{\partial \xi} \end{bmatrix}$$

$$\det[J] = \det \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta} =$$

$$= \frac{\partial([N(\xi, \eta)]\{x_i\}_e)}{\partial \xi} \frac{\partial([N(\xi, \eta)]\{y_i\}_e)}{\partial \eta} - \frac{\partial([N(\xi, \eta)]\{y_i\}_e)}{\partial \xi} \frac{\partial([N(\xi, \eta)]\{x_i\}_e)}{\partial \eta} =$$

$$= \left(\frac{\partial[N(\xi, \eta)]}{\partial \xi} \{x_i\}_e \right) \left(\frac{\partial[N(\xi, \eta)]}{\partial \eta} \{y_i\}_e \right) - \left(\frac{\partial[N(\xi, \eta)]}{\partial \xi} \{y_i\}_e \right) \left(\frac{\partial[N(\xi, \eta)]}{\partial \eta} \{x_i\}_e \right)$$

$\begin{matrix} 1 \times 4 & & 4 \times 1 & & 1 \times 4 & & 4 \times 1 & & 1 \times 4 & & 4 \times 1 \end{matrix}$

$$\frac{\partial \xi}{\partial x} = \frac{1}{\det[J]} \frac{\partial y}{\partial \eta} = \frac{1}{\det[J]} \frac{\partial([N(\xi, \eta)]\{y_i\}_e)}{\partial \eta} = \frac{1}{\det[J]} \frac{\partial[N(\xi, \eta)]}{\partial \eta} \{y_i\}_e$$

$\begin{matrix} 1 \times 4 & & 4 \times 1 \end{matrix}$

$$\frac{\partial \eta}{\partial x} = -\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} = -\frac{1}{\det[J]} \frac{\partial([N(\xi, \eta)]\{y_i\}_e)}{\partial \xi} = -\frac{1}{\det[J]} \frac{\partial[N(\xi, \eta)]}{\partial \xi} \{y_i\}_e$$

$\begin{matrix} 1 \times 4 & & 4 \times 1 \end{matrix}$

$$\frac{\partial \xi}{\partial y} = -\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} = -\frac{1}{\det[J]} \frac{\partial([N(\xi, \eta)]\{x_i\}_e)}{\partial \eta} = -\frac{1}{\det[J]} \frac{\partial[N(\xi, \eta)]}{\partial \eta} \{x_i\}_e$$

$\begin{matrix} 1 \times 4 & & 4 \times 1 \end{matrix}$

$$\frac{\partial \eta}{\partial y} = \frac{1}{\det[J]} \frac{\partial x}{\partial \xi} = \frac{1}{\det[J]} \frac{\partial([N(\xi, \eta)]\{x_i\}_e)}{\partial \xi} = \frac{1}{\det[J]} \frac{\partial[N(\xi, \eta)]}{\partial \xi} \{x_i\}_e$$

$\begin{matrix} 1 \times 4 & & 4 \times 1 \end{matrix}$

$$\left[\frac{\partial N}{\partial \xi} \right]_{2 \times 8} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\ 0 & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} \end{bmatrix}$$

$$\left[\frac{\partial N}{\partial \eta} \right]_{2 \times 8} = \begin{bmatrix} \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 \\ 0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} \end{bmatrix}$$

$$[D]_{3 \times 3} = \frac{E}{(1 - \nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1 - \nu) \end{bmatrix}$$

macierz
konstrytutywna dla PSN:

macierz sztywności elementu:

$$[k]_e_{8 \times 8} = t_e \int_{-1}^1 \int_{-1}^1 [B(\xi, \eta)]_{8 \times 3}^T [D]_{3 \times 3} [B(\xi, \eta)]_{3 \times 8} \det[J] d\xi d\eta$$

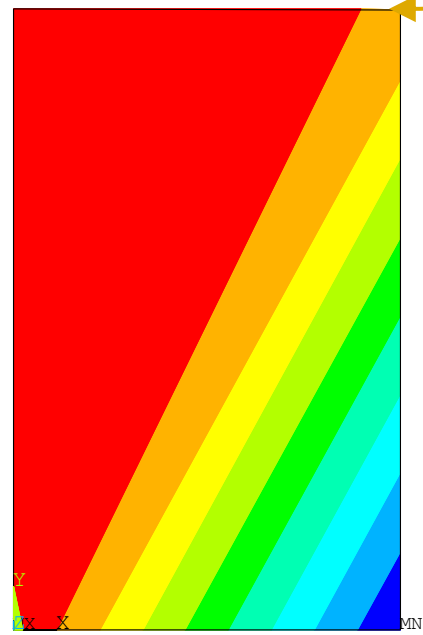
Całkowane numerycznie mieszaną regułą kwadratur
aby uniknąć „*shear locking*”
(Enhanced strain formulation)

$$\{q\} = [k]^{-1} \{F\}$$

4×1 4×4 4×1

F		q	
0		-.012724	u2
0		-.001562	u3
1000	N	.02286	v3
2000	N	.045711	v4

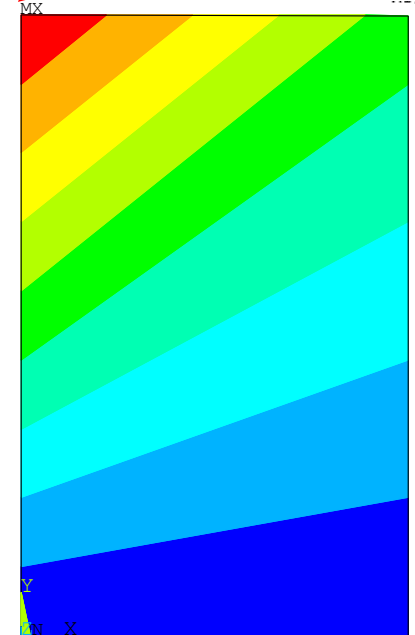
UX displacement



PLOT NO. 1
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMN =-.012724

Blue	-.012724
Light Blue	-.01131
Cyan	-.009896
Green	-.008483
Light Green	-.007069
Yellow-Green	-.005655
Yellow	-.004241
Orange	-.002828
Red	-.001414
Dark Red	0

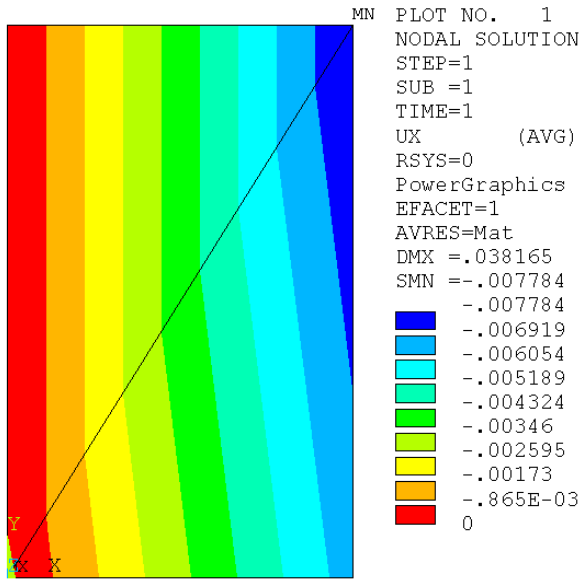
UY displacement



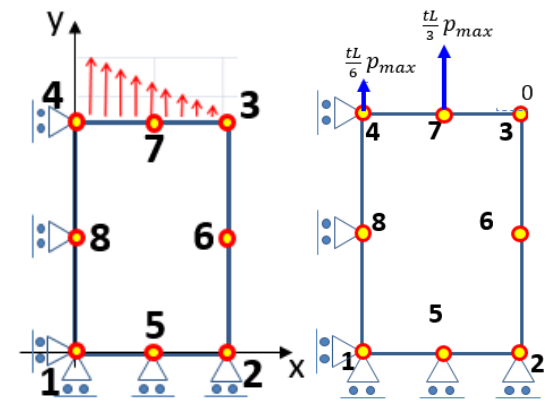
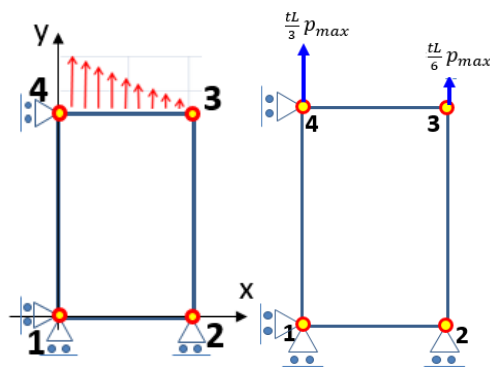
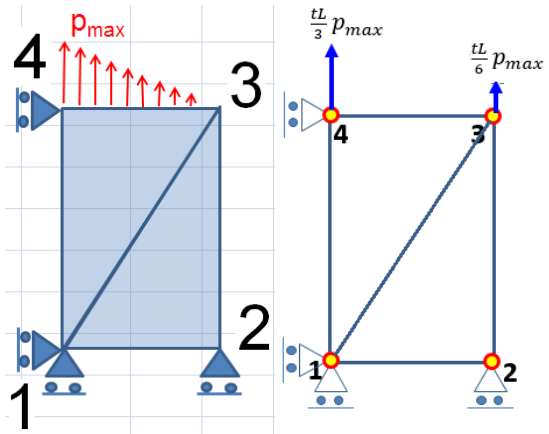
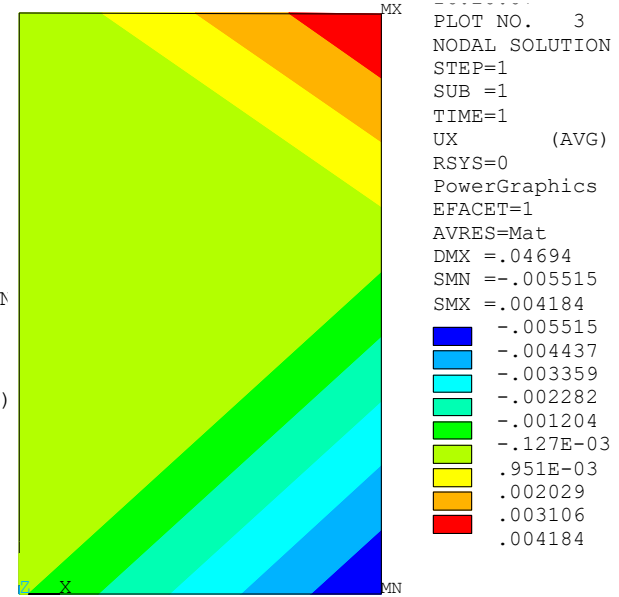
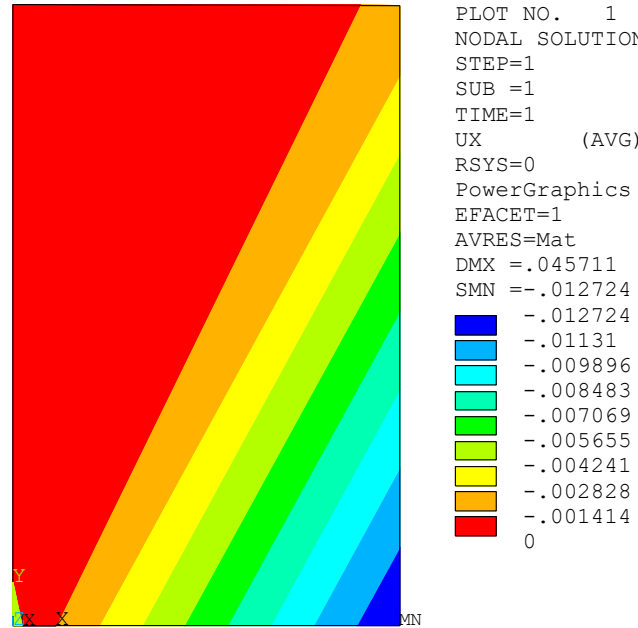
PLOT NO. 2
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMX =.045711

Blue	0
Light Blue	.005079
Cyan	.010158
Green	.015237
Light Green	.020316
Yellow-Green	.025395
Yellow	.030474
Orange	.035553
Red	.040632
Dark Red	.045711

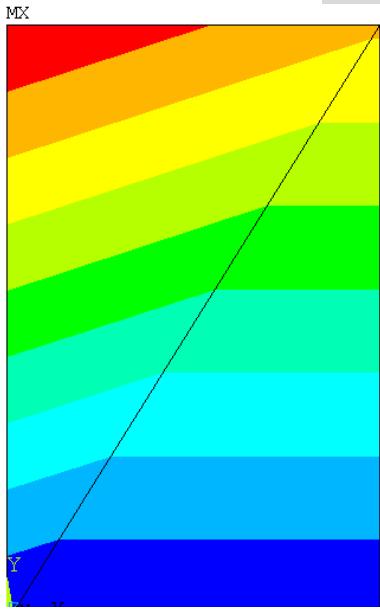
Przemieszczenia na kierunek X



UX displacement



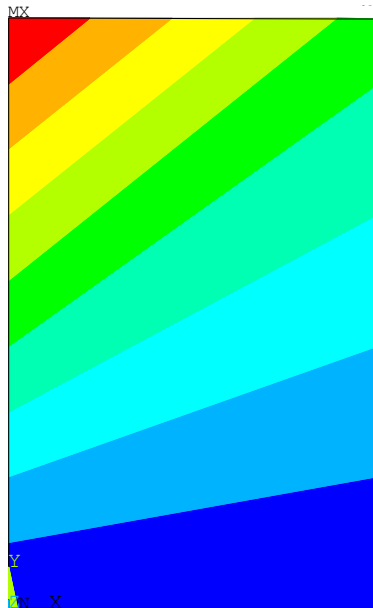
Przemieszczenia na kierunku Y



PLOT NO. 2
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.038165
 SMX =.038165

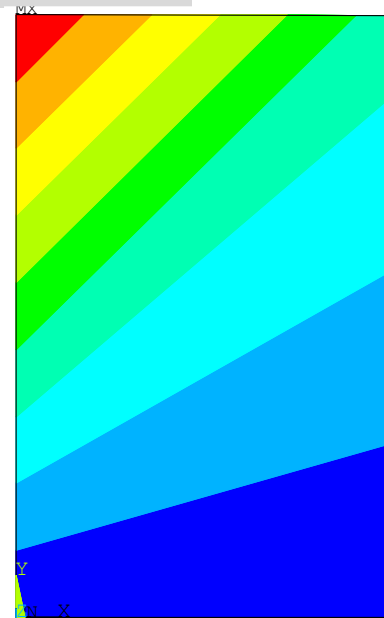
0	.004241
	.008481
	.012722
	.016962
	.021203
	.025444
	.029684
	.033925
	.038165

UY displacement



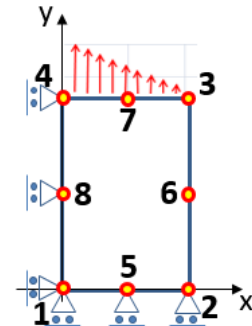
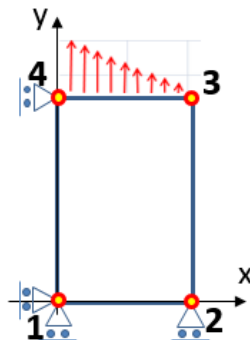
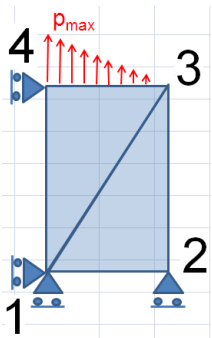
PLOT NO. 2
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMX =.045711

0	.005079
	.010158
	.015237
	.020316
	.025395
	.030474
	.035553
	.040632
	.045711

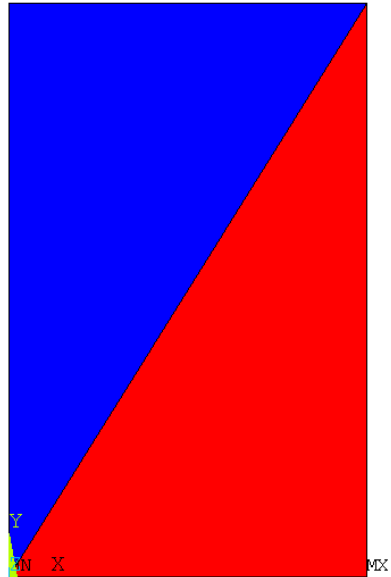


PLOT NO. 4
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.04694
 SMX =.04694

0	.005216
	.010431
	.015647
	.020862
	.026078
	.031293
	.036509
	.041724
	.04694



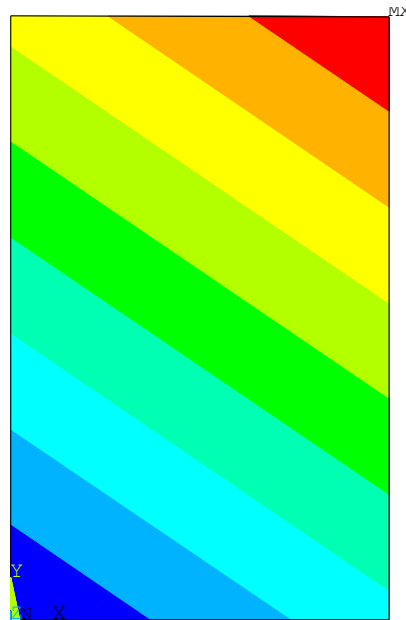
Odształcenia na kierunku X



PLOT NO. 7
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.038165
 SMN =-.156E-03
 SMX =-.130E-03

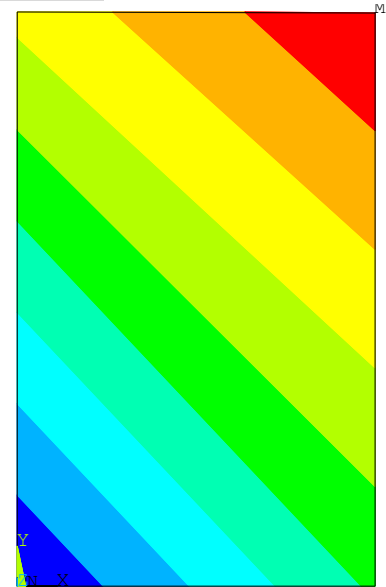
Blue	-.156E-03
Light Blue	-.153E-03
Cyan	-.150E-03
Green	-.147E-03
Light Green	-.144E-03
Yellow-Green	-.141E-03
Yellow	-.139E-03
Orange	-.136E-03
Red-Orange	-.133E-03
Red	-.130E-03

ϵ_x strain



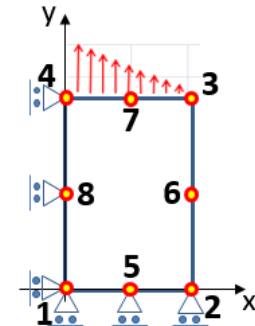
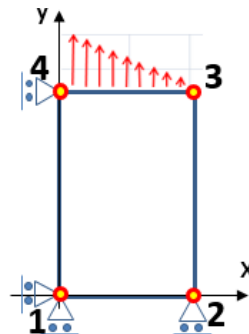
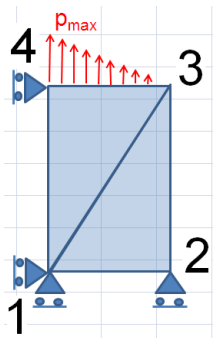
PLOT NO. 3
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMN =-.302E-03
 SMX =.164E-04

Blue	-.302E-03
Light Blue	-.267E-03
Cyan	-.231E-03
Green	-.196E-03
Light Green	-.161E-03
Yellow-Green	-.125E-03
Yellow	-.898E-04
Orange	-.544E-04
Red-Orange	-.190E-04
Red	.164E-04



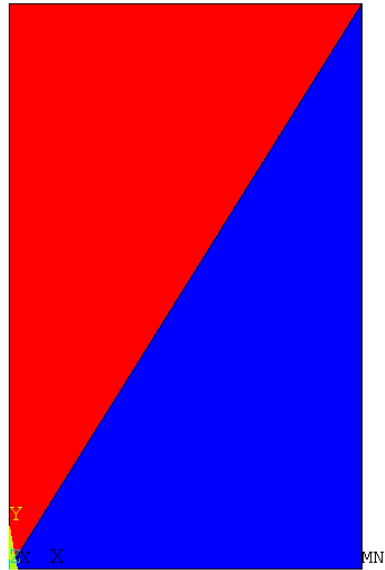
PLOT NO. 12
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.04694
 SMN =-.313E-03
 SMX =.157E-05

Blue	-.313E-03
Light Blue	-.278E-03
Cyan	-.243E-03
Green	-.208E-03
Light Green	-.173E-03
Yellow-Green	-.138E-03
Yellow	-.103E-03
Orange	-.682E-04
Red-Orange	-.333E-04
Red	.157E-05

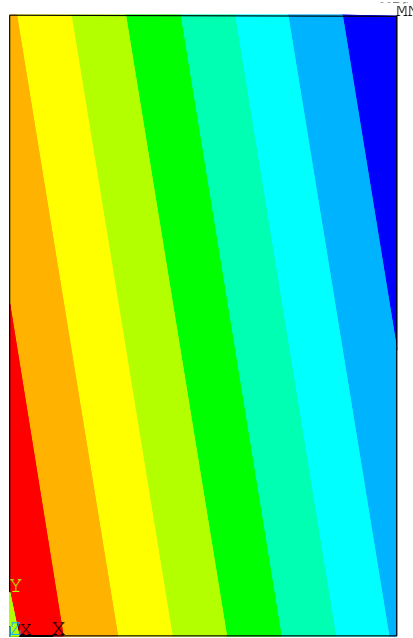


Odkształcenia na kierunku Y

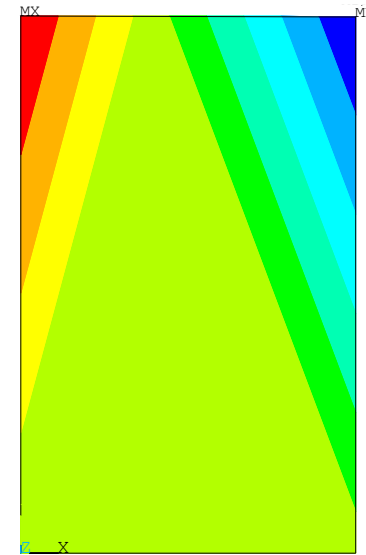
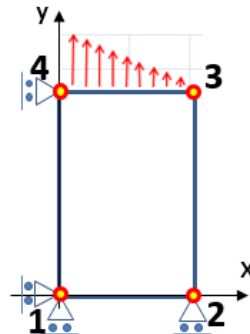
PLOT NO. 8
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.038165
 SMN =.380E-03
 SMX =.477E-03
 .380E-03
 .391E-03
 .402E-03
 .412E-03
 .423E-03
 .434E-03
 .445E-03
 .456E-03
 .466E-03
 .477E-03



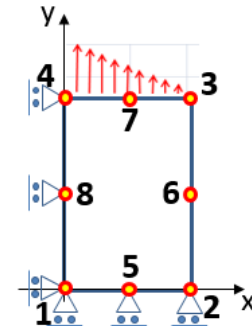
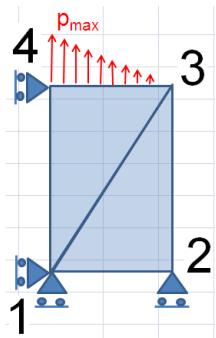
ϵ_y strain



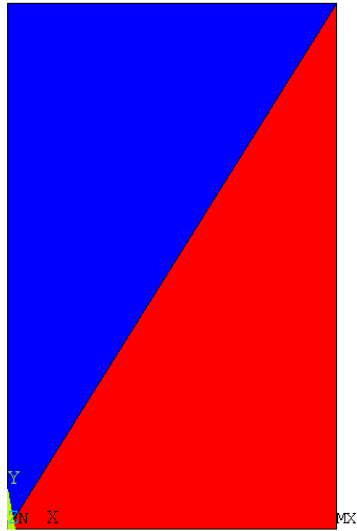
PLOT NO. 4
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMN =.249E-03
 SMX =.609E-03
 .249E-03
 .289E-03
 .329E-03
 .369E-03
 .409E-03
 .449E-03
 .489E-03
 .529E-03
 .569E-03
 .609E-03



PLOT NO. 13
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.04694
 SMN =.238E-04
 SMX =.769E-03
 .238E-04
 .107E-03
 .189E-03
 .272E-03
 .355E-03
 .438E-03
 .520E-03
 .603E-03
 .686E-03
 .769E-03



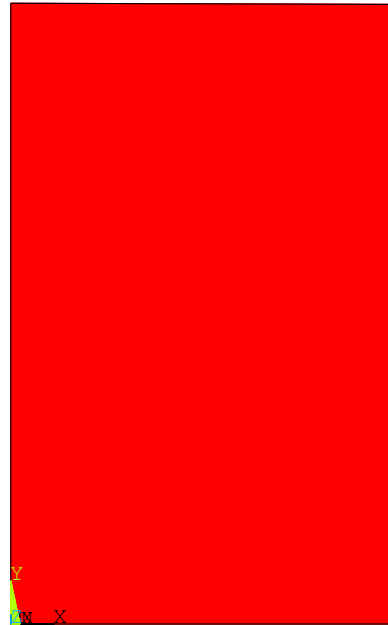
Odkształcenia postaciowe



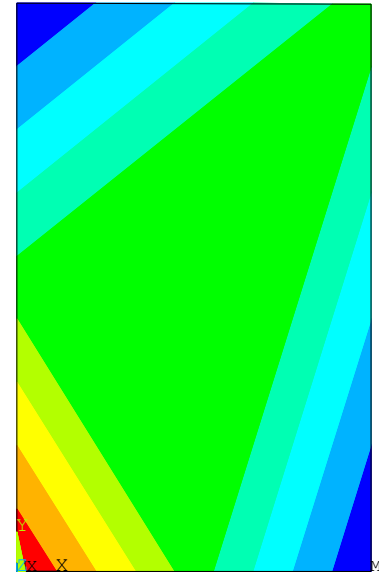
PLOT NO. 1
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.038165
 SMN =-.155E-03
 SMX =-.160E-04

Blue	-.140E-03
Light Blue	-.124E-03
Cyan	-.109E-03
Green	-.933E-04
Light Green	-.779E-04
Yellow	-.624E-04
Orange	-.470E-04
Red	-.315E-04
Dark Red	-.160E-04

γ_{xy} strain

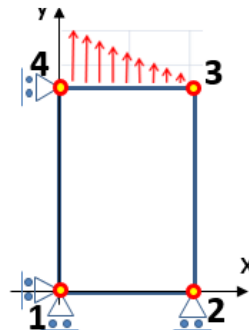
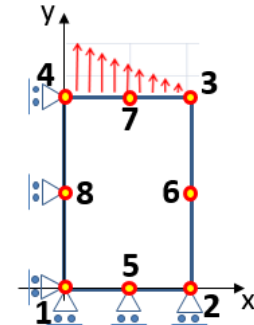
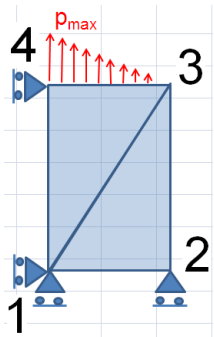


PLOT NO. 5
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELXY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMN =-.159E-03
 SMX =-.159E-03



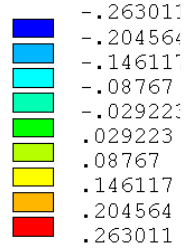
PLOT NO. 14
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 EPELXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.04694
 SMN =-.259E-03
 SMX =.105E-03

Blue	-.259E-03
Light Blue	-.219E-03
Cyan	-.178E-03
Green	-.138E-03
Light Green	-.974E-04
Yellow	-.569E-04
Orange	-.164E-04
Red	.241E-04
Dark Red	.646E-04
Dark Red	.105E-03

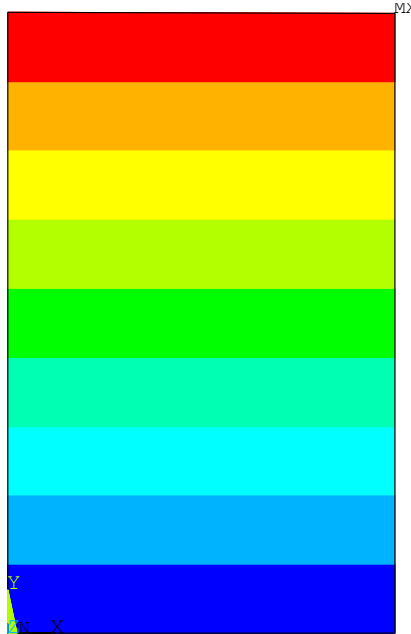


Naprężenia na kierunku X

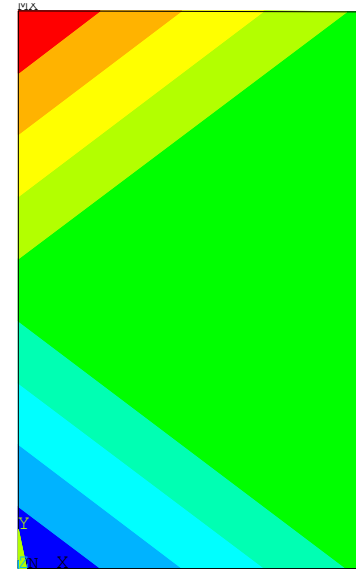
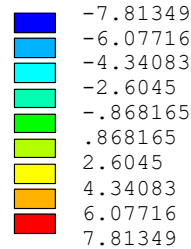
PLOT NO. 3
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.038165
 SMN =-.263011
 SMX =.263011



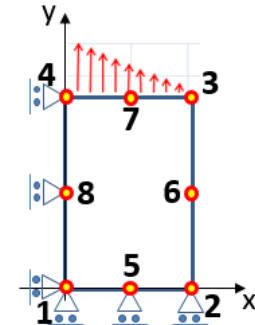
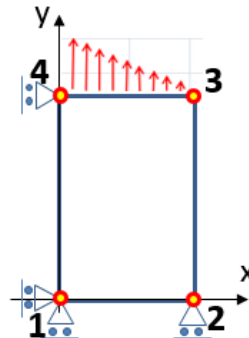
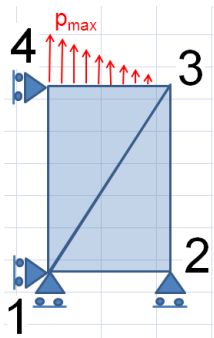
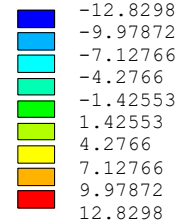
σ_x stress



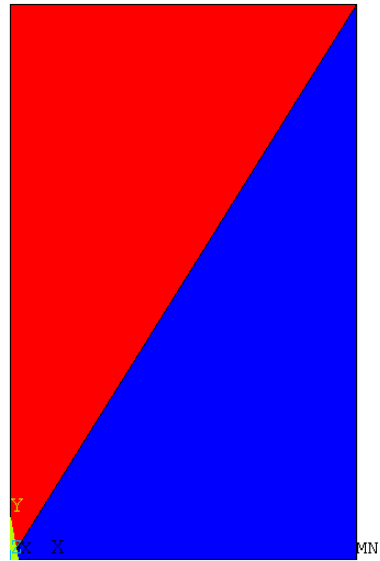
PLOT NO. 6
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.045711
 SMN =-7.81349
 SMX =7.81349



PLOT NO. 5
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.04694
 SMN =-12.8298
 SMX =12.8298



Naprężenia na kierunek Y

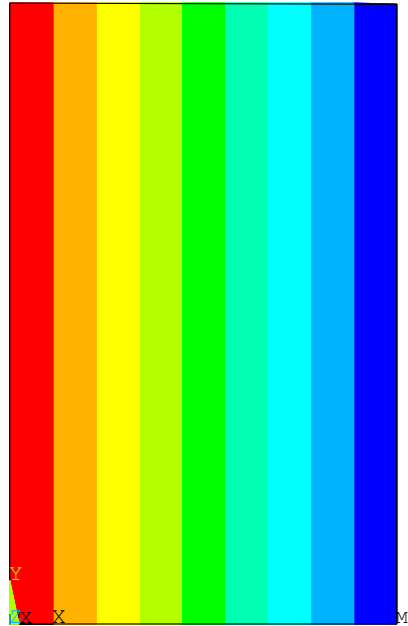


PLOT NO. 4
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1

DMX =.038165
 SMN =26.518
 SMX =33.482

26.518
27.292
28.065
28.839
29.613
30.387
31.161
31.935
32.708
33.482

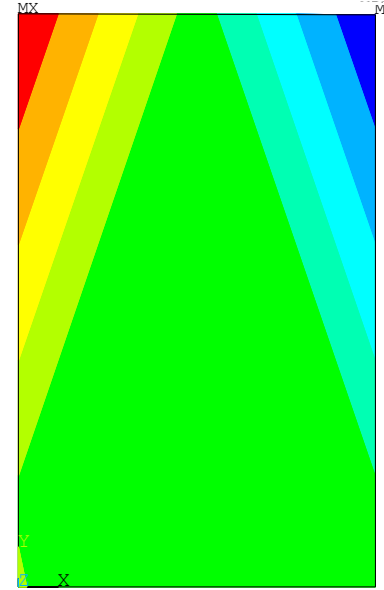
σ_y stress



PLOT NO. 7
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1

DMX =.045711
 SMN =20.0025
 SMX =39.9975

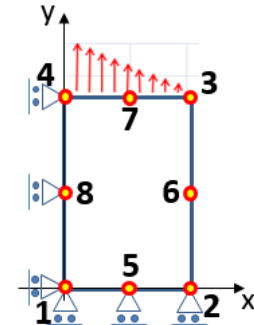
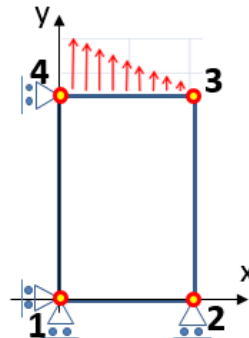
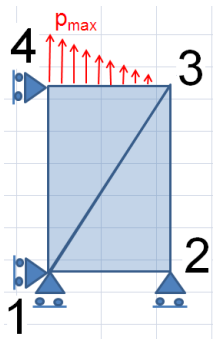
20.0025
22.2242
24.4459
26.6675
28.8892
31.1108
33.3325
35.5541
37.7758
39.9975



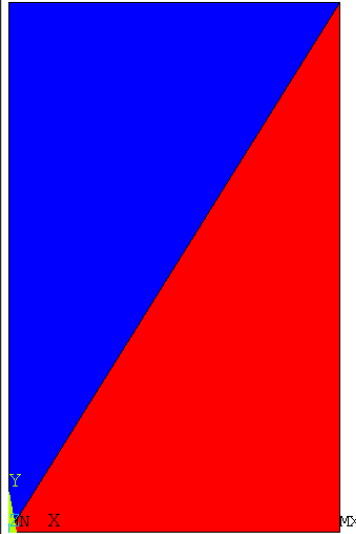
PLOT NO. 6
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1

DMX =.04694
 SMN =1.91405
 SMX =58.086

1.91405
8.15537
14.3967
20.638
26.8793
33.1207
39.362
45.6033
51.8446
58.086

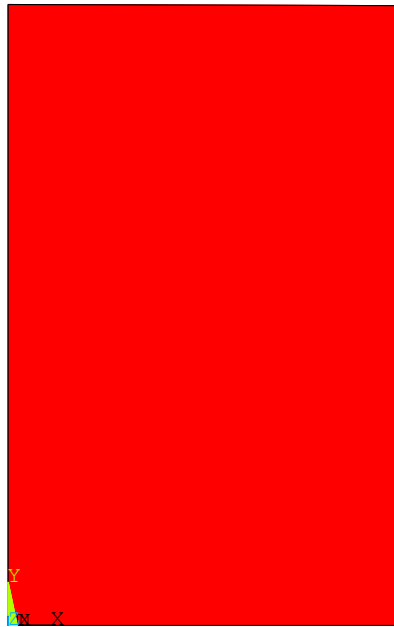


Naprężenia styczne

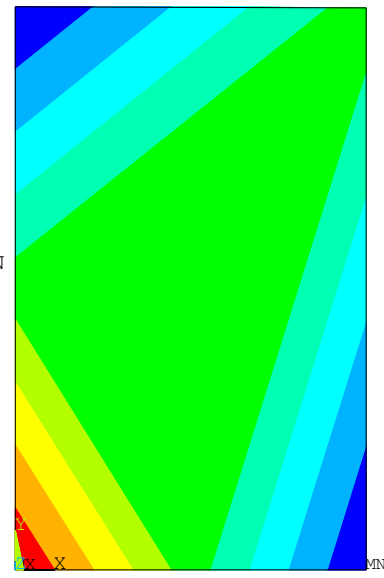


PLOT NO. 5
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.038165
 SMN =-4.074
 SMX =-.420818
 -4.074
 -3.668
 -3.262
 -2.856
 -2.45
 -2.044
 -1.638
 -1.233
 -.826679
 -.420818

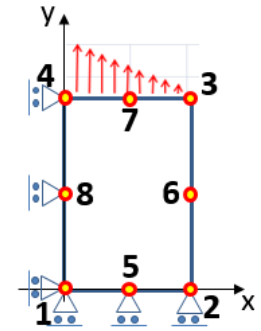
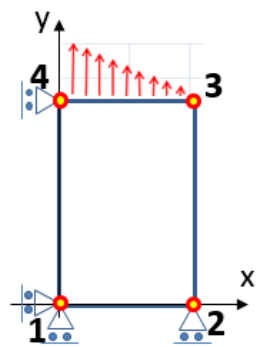
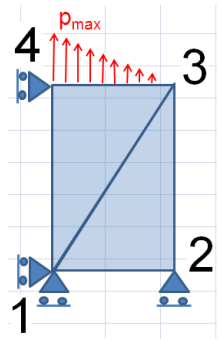
τ_{xy} stress



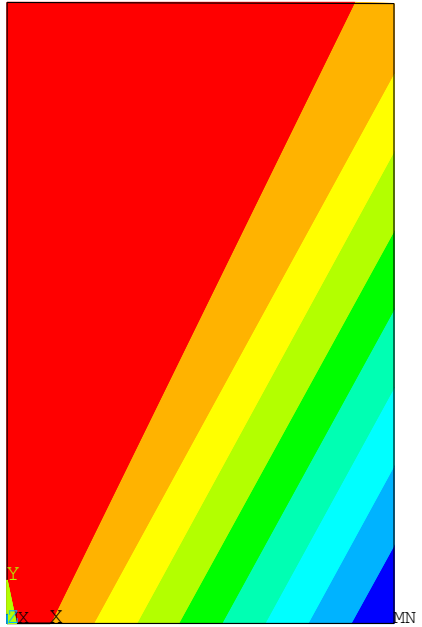
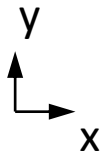
PLOT NO. 8
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.045711
 SMN =-4.16719
 SMX =-4.16719



PLOT NO. 7
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.04694
 SMN =-6.81074
 SMX =2.7595
 -6.81074
 -5.74738
 -4.68402
 -3.62066
 -2.5573
 -1.49394
 -.430576
 .632784
 1.69614
 2.7595



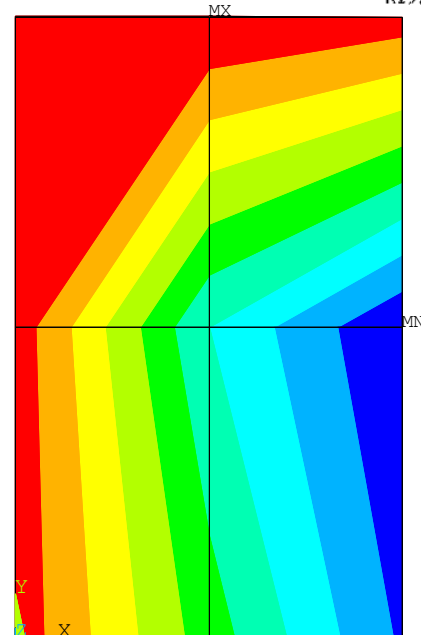
Wpływ dyskretyzacji na jakość wyników



PLOT NO. 1
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMN =-.012724

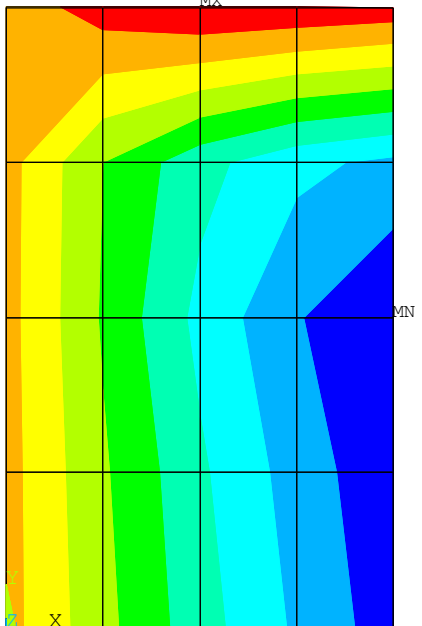
Blue	-.012724
Light Blue	-.01131
Cyan	-.009896
Green	-.008483
Light Green	-.007069
Yellow	-.005655
Orange	-.004241
Red	-.002828
Dark Red	-.001414
Black	0

UX
 [mm]



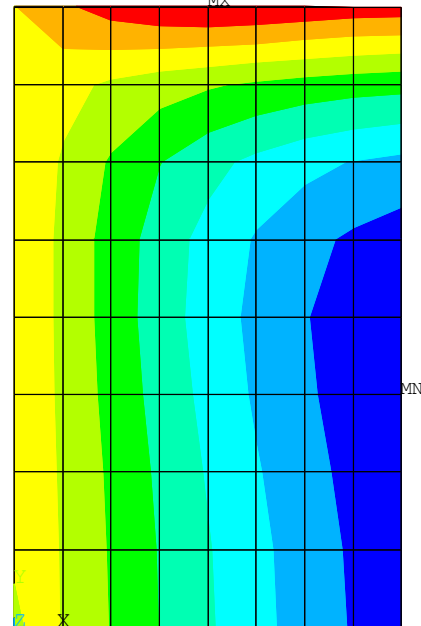
PLOT NO. 10
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046436
 SMN =-.009823

Blue	-.009823
Light Blue	-.008682
Cyan	-.00754
Green	-.006398
Light Green	-.005257
Yellow	-.004115
Orange	-.002974
Red	-.001832
Dark Red	-.690E-03
Black	.451E-03



PLOT NO. 15
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046861
 SMN =-.009151

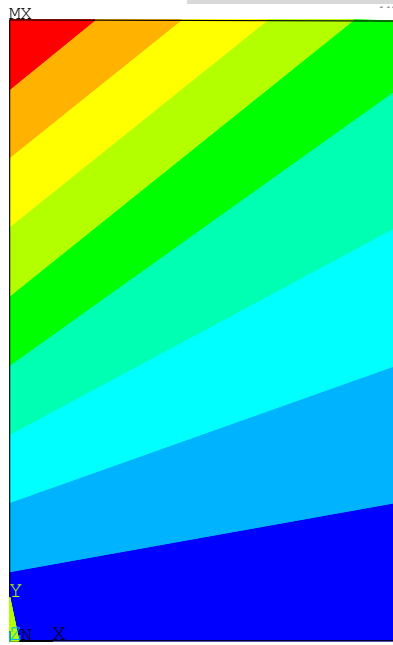
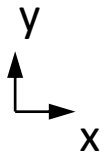
Blue	-.009151
Light Blue	-.007909
Cyan	-.006666
Green	-.005424
Light Green	-.004181
Yellow	-.002938
Orange	-.001696
Red	-.453E-03
Dark Red	.789E-03
Black	.002032



PLOT NO. 23
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047071
 SMN =-.008957

Blue	-.008957
Light Blue	-.007671
Cyan	-.006385
Green	-.005099
Light Green	-.003813
Yellow	-.002527
Orange	-.001241
Red	.452E-04
Dark Red	.001331
Black	.002617

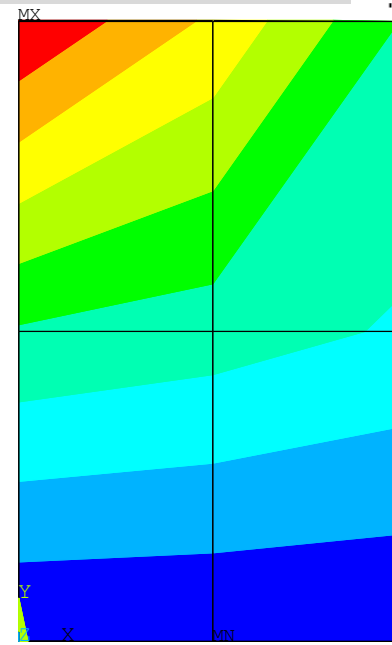
Wpływ dyskretyzacji na jakość wyników



00:00:10
 PLOT NO. 2
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMX =.045711

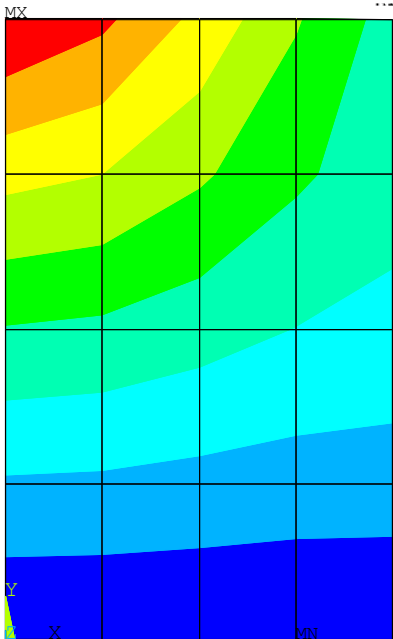
0
.005079
.010158
.015237
.020316
.025395
.030474
.035553
.040632
.045711

UY
 [mm]



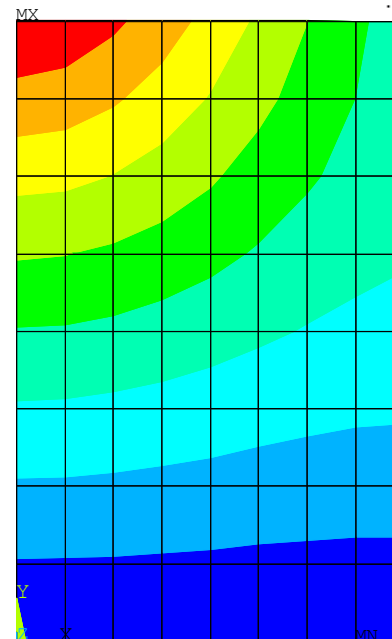
00:00:23
 PLOT NO. 11
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046436
 SMX =.046436

0
.00516
.010319
.015479
.020638
.025798
.030957
.036117
.041276
.046436



00:00:21
 PLOT NO. 16
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046861
 SMX =.046861

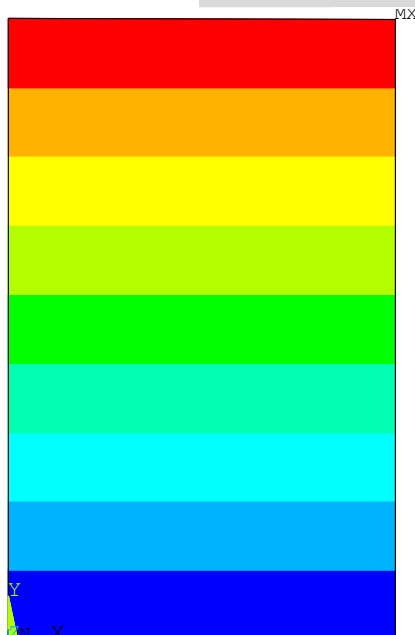
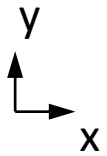
0
.005207
.010414
.01562
.020827
.026034
.031241
.036448
.041654
.046861



00:00:19
 PLOT NO. 24
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047071
 SMX =.047071

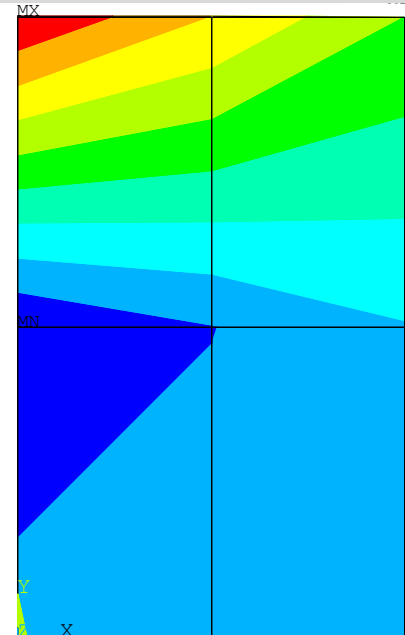
0
.00523
.01046
.01569
.02092
.026151
.031381
.036611
.041841
.047071

Wpływ dyskretyzacji na jakość wyników

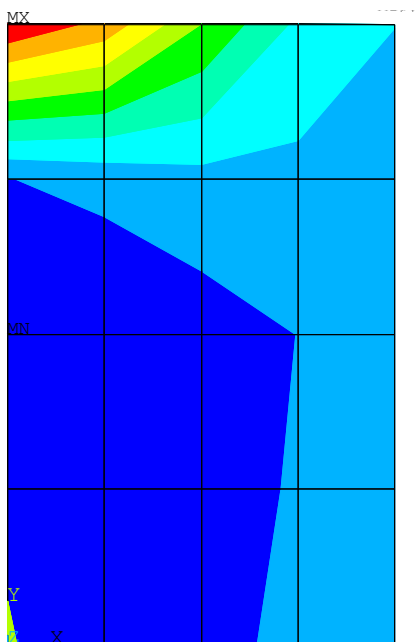


PLOT NO. 6
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.045711
 SMN =-7.81349
 SMX =7.81349
 -7.81349
 -6.07716
 -4.34083
 -2.6045
 -.868165
 .868165
 2.6045
 4.34083
 6.07716
 7.81349

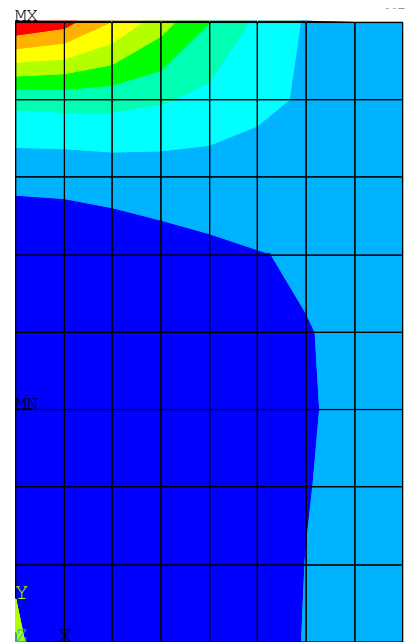
σ_x
 [MPa]
 NODAL
 SOLUTION



PLOT NO. 31
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046436
 SMN =-5.95909
 SMX =14.5388
 -5.95909
 -3.68154
 -1.404
 .873548
 3.15109
 5.42864
 7.70619
 9.98373
 12.2613
 14.5388

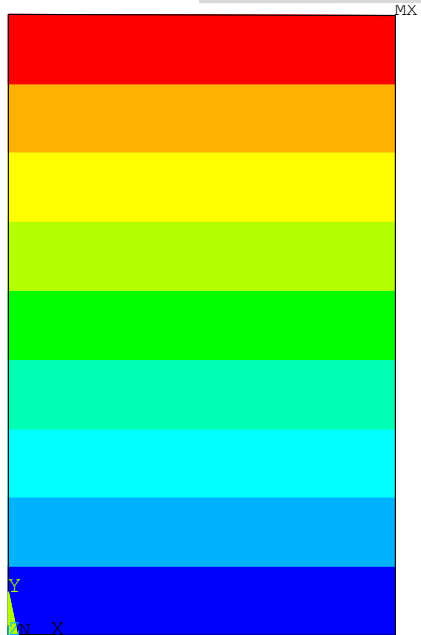
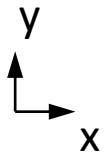


PLOT NO. 20
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046861
 SMN =-4.63339
 SMX =24.1699
 -4.63339
 -1.43302
 1.76735
 4.96772
 8.16809
 11.3685
 14.5688
 17.7692
 20.9696
 24.1699



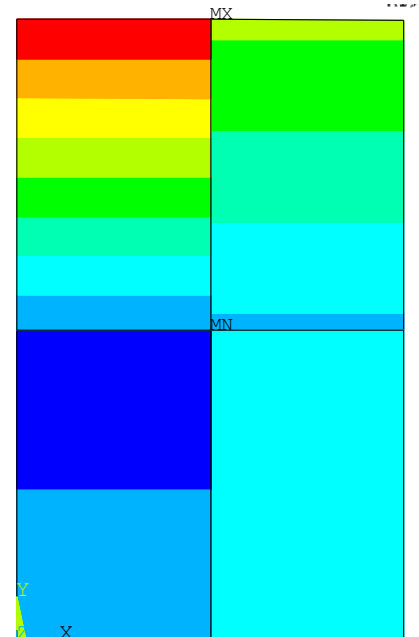
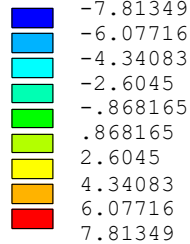
PLOT NO. 25
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047071
 SMN =-4.69728
 SMX =30.1917
 -4.69728
 -.820734
 3.05582
 6.93237
 10.8089
 14.6855
 18.562
 22.4386
 26.3151
 30.1917

Wpływ dyskretyzacji na jakość wyników

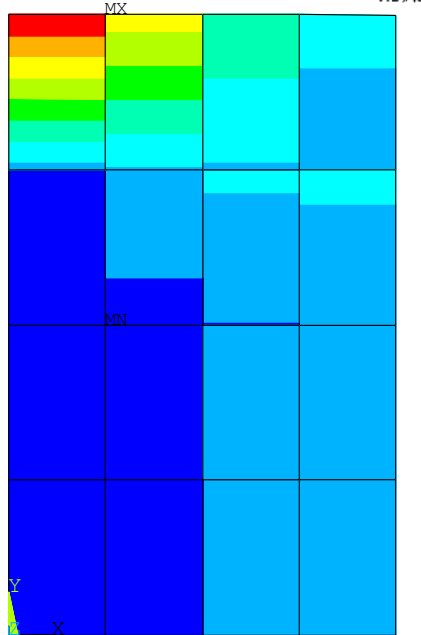
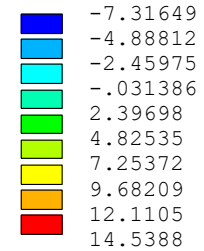


00:00:06
 PLOT NO. 6
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.045711
 SMN =-7.81349
 SMX =7.81349

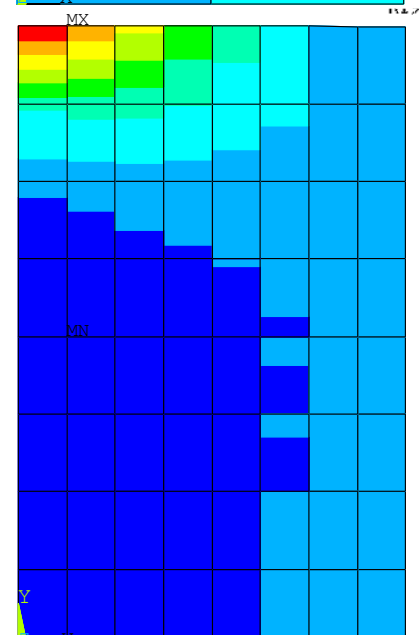
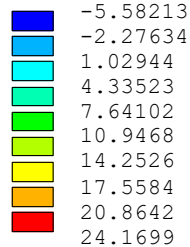
σ_x
 [MPa]
 ELEMENT SOLUTION



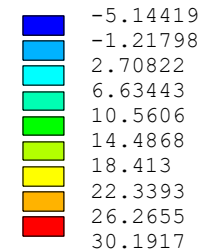
00:00:34
 PLOT NO. 12
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.046436
 SMN =-7.31649
 SMX =14.5388



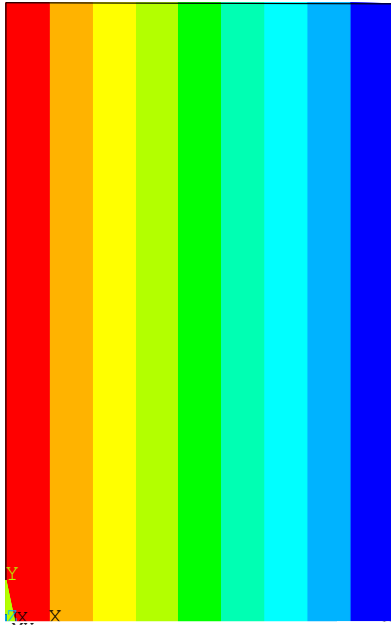
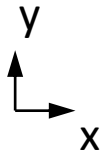
00:00:31
 PLOT NO. 17
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.046861
 SMN =-5.58213
 SMX =24.1699



00:00:40
 PLOT NO. 28
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SX (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.047071
 SMN =-5.14419
 SMX =30.1917



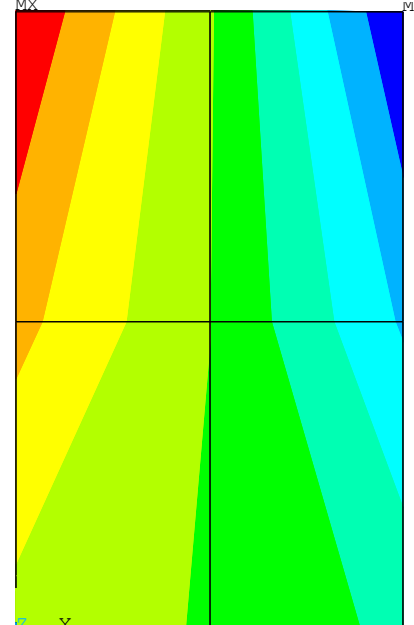
Wpływ dyskretyzacji na jakość wyników



PLOT NO. 7
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.045711
 SMN =20.0025
 SMX =39.9975

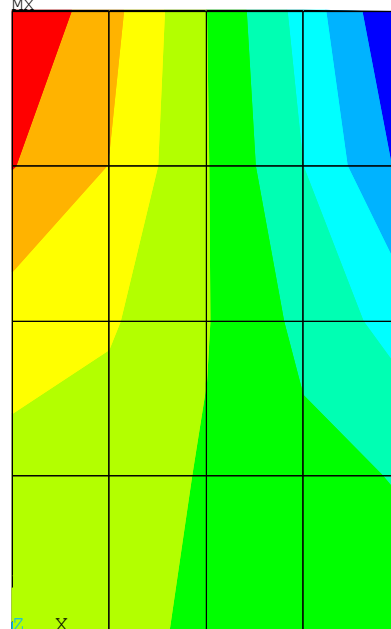
σ_y
 [MPa]
 NODAL
 SOLUTION

- 20.0025
- 22.2242
- 24.4459
- 26.6675
- 28.8892
- 31.1108
- 33.3325
- 35.5541
- 37.7758
- 39.9975



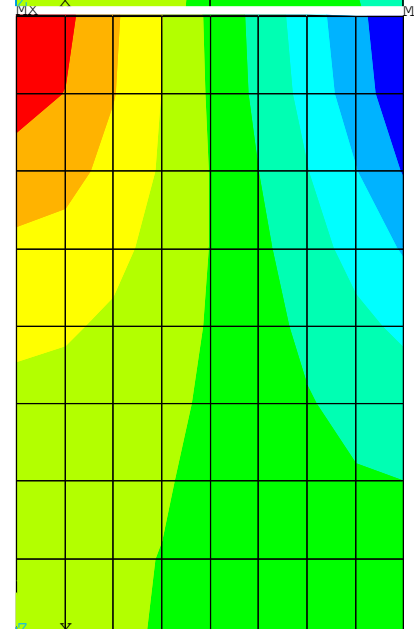
PLOT NO. 32
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046436
 SMN =9.68398
 SMX =47.7465

- 9.68398
- 13.9132
- 18.1423
- 22.3715
- 26.6007
- 30.8298
- 35.059
- 39.2882
- 43.5173
- 47.7465



PLOT NO. 21
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046861
 SMN =3.04724
 SMX =52.8834

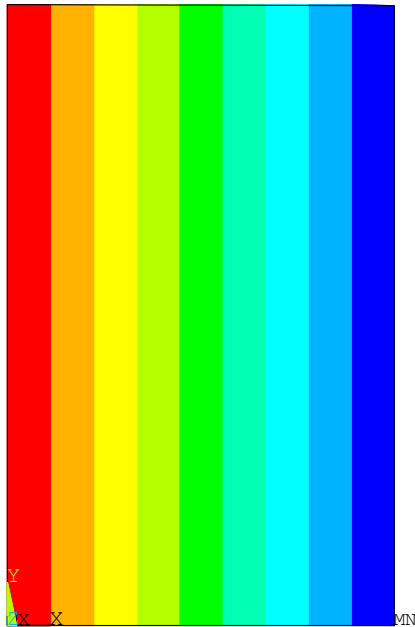
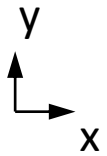
- 3.04724
- 8.5846
- 14.122
- 19.6593
- 25.1967
- 30.734
- 36.2714
- 41.8087
- 47.3461
- 52.8834



PLOT NO. 26
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047071
 SMN =.427497
 SMX =56.1359

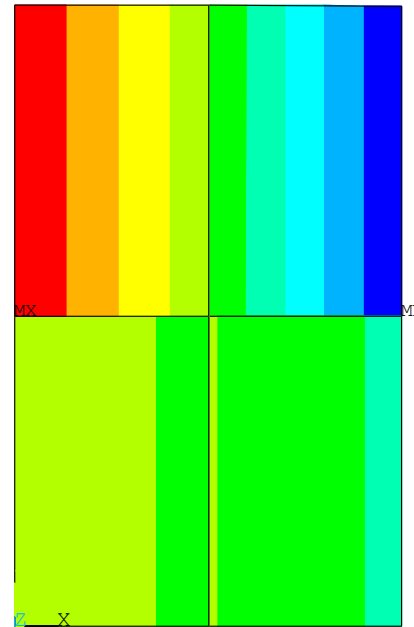
- .427497
- 6.61732
- 12.8071
- 18.997
- 25.1868
- 31.3766
- 37.5664
- 43.7562
- 49.946
- 56.1359

Wpływ dyskretyzacji na jakość wyników

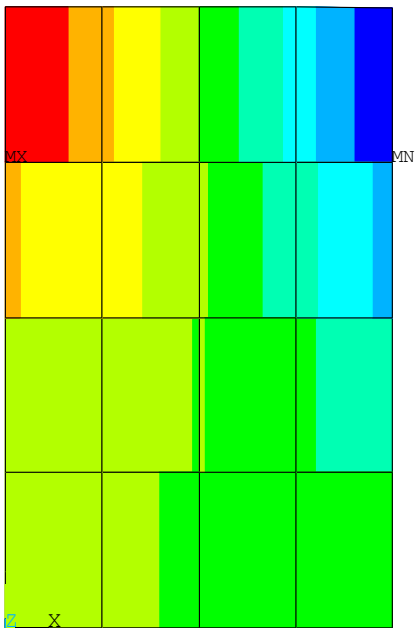


PLOT NO. 7
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.045711
 SMN =20.0025
 SMX =39.9975
 20.0025
 22.2242
 24.4459
 26.6675
 28.8892
 31.1108
 33.3325
 35.5541
 37.7758
 39.9975

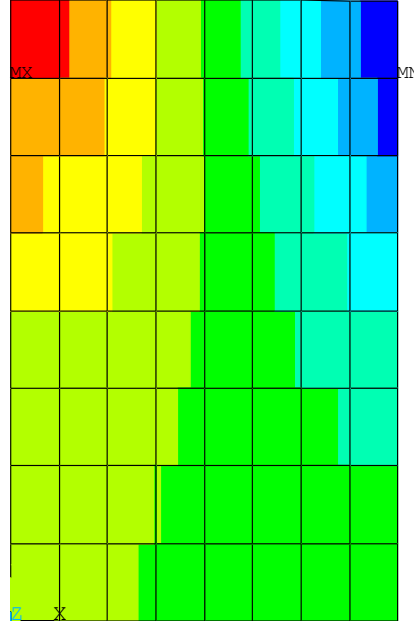
σ_y
 [MPa]
 ELEMENT SOLUTION



PLOT NO. 13
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.046436
 SMN =9.68398
 SMX =47.7465
 9.68398
 13.9132
 18.1423
 22.3715
 26.6007
 30.8298
 35.059
 39.2882
 43.5173
 47.7465

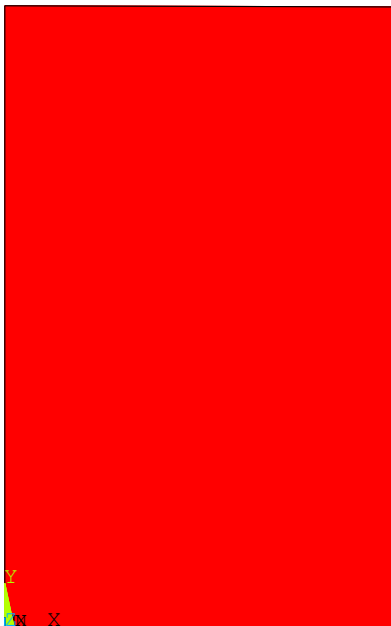
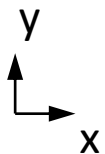


PLOT NO. 18
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.046861
 SMN =3.04724
 SMX =52.8834
 3.04724
 8.5846
 14.122
 19.6593
 25.1967
 30.734
 36.2714
 41.8087
 47.3461
 52.8834



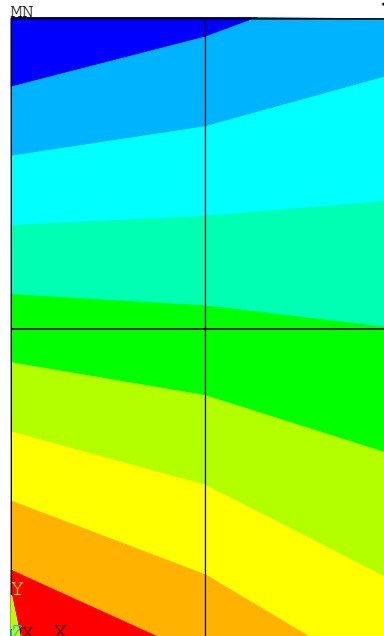
PLOT NO. 29
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.047071
 SMN =.427497
 SMX =56.1359
 .427497
 6.61732
 12.8071
 18.997
 25.1868
 31.3766
 37.5664
 43.7562
 49.946
 56.1359

Wpływ dyskretyzacji na jakość wyników



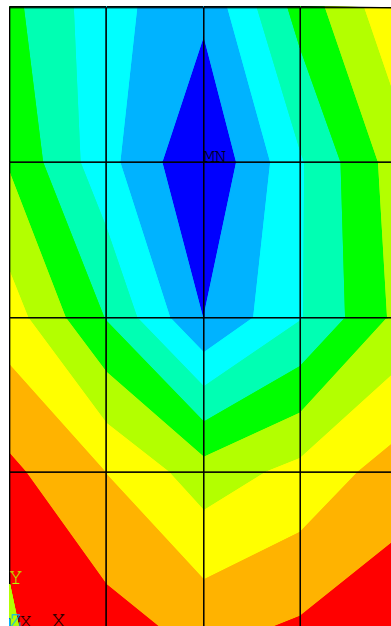
PLOT NO. 8
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.045711
 SMN =-4.16719
 SMX =-4.16719

τ_{xy}
 [MPa]
NODAL SOLUTION



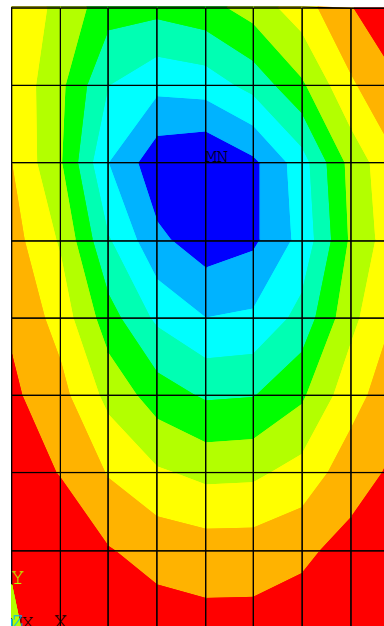
PLOT NO. 33
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046436
 SMN =-4.70816
 SMX =-1.65009

Blue	-4.70816
Light Blue	-4.36837
Cyan	-4.02859
Green	-3.6888
Light Green	-3.34902
Yellow-Green	-3.00923
Yellow	-2.66944
Orange	-2.32966
Red-Orange	-1.98987
Red	-1.65009



PLOT NO. 22
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.046861
 SMN =-5.49877
 SMX =-.425279

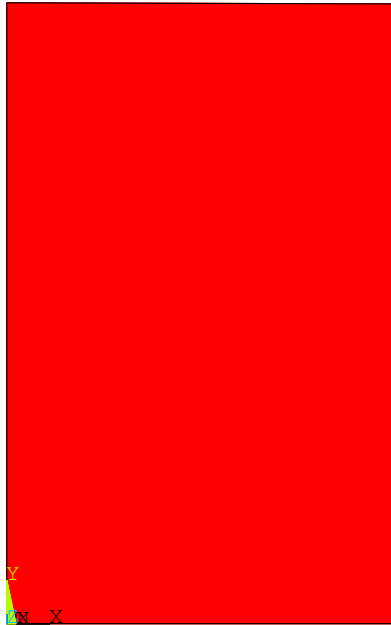
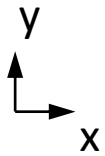
Blue	-5.49877
Light Blue	-4.93504
Cyan	-4.37132
Green	-3.8076
Light Green	-3.24388
Yellow-Green	-2.68016
Yellow	-2.11644
Orange	-1.55272
Red-Orange	-.989
Red	-.425279



PLOT NO. 27
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.047071
 SMN =-6.68
 SMX =-.106963

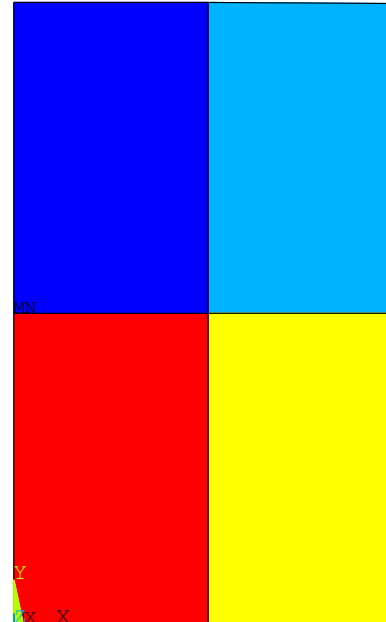
Blue	-6.68
Light Blue	-5.94966
Cyan	-5.21933
Green	-4.48899
Light Green	-3.75865
Yellow-Green	-3.02831
Yellow	-2.29798
Orange	-1.56764
Red-Orange	-.837301
Red	-.106963

Wpływ dyskretyzacji na jakość wyników



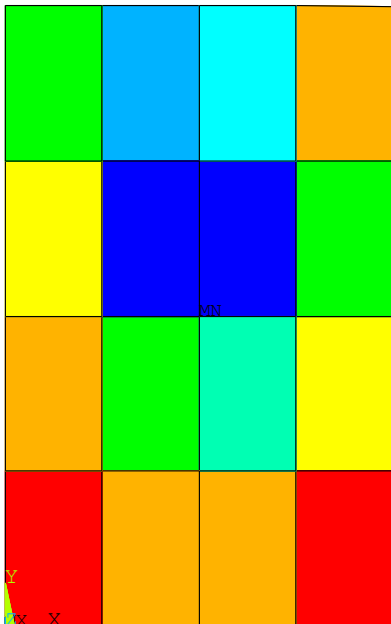
PLOT NO. 8
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.045711
 SMN =-4.16719
 SMX =-4.16719

τ_{xy}
 [MPa]
 ELEMENT SOLUTION



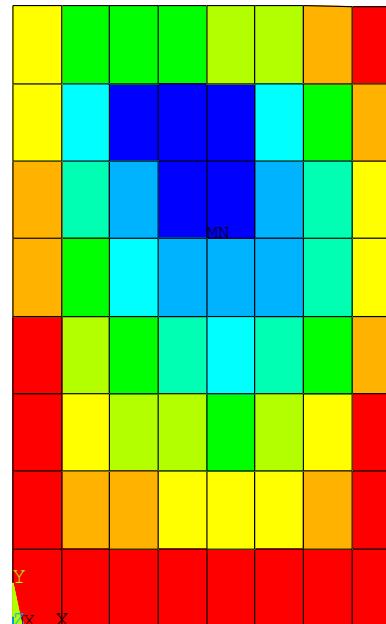
PLOT NO. 14
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.046436
 SMN =-4.70816
 SMX =-1.65009

Blue	-4.70816
Light Blue	-4.36837
Cyan	-4.02859
Light Green	-3.6888
Green	-3.34902
Yellow-Green	-3.00923
Yellow	-2.66944
Orange	-2.32966
Light Orange	-1.98987
Red	-1.65009



PLOT NO. 19
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.046861
 SMN =-6.33246
 SMX =-.425279

Blue	-6.33246
Light Blue	-5.6761
Cyan	-5.01975
Light Green	-4.3634
Green	-3.70704
Yellow-Green	-3.05069
Yellow	-2.39434
Orange	-1.73799
Light Orange	-1.08163
Red	-.425279



PLOT NO. 30
 ELEMENT SOLUTION
 STEP=1
 SUB =1
 TIME=1
 SXY (NOAVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 DMX =.047071
 SMN =-6.83544
 SMX =-.106963

Blue	-6.83544
Light Blue	-6.08783
Cyan	-5.34022
Light Green	-4.59261
Green	-3.84501
Yellow-Green	-3.0974
Yellow	-2.34979
Orange	-1.60218
Light Orange	-.854572
Red	-.106963