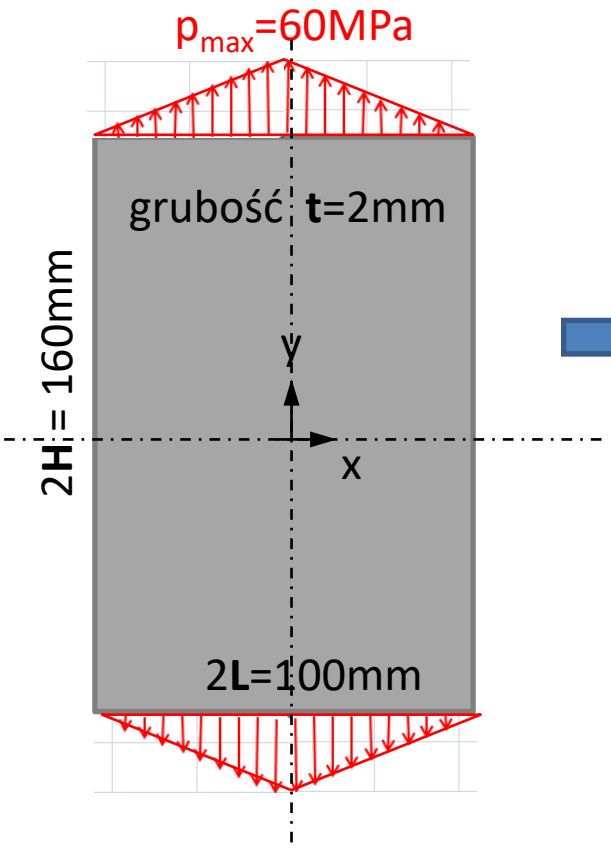
Wydział Mechaniczny Energetyki i Lotnictwa
Zakład Wytrzymałości Materiałów i Konstrukcji

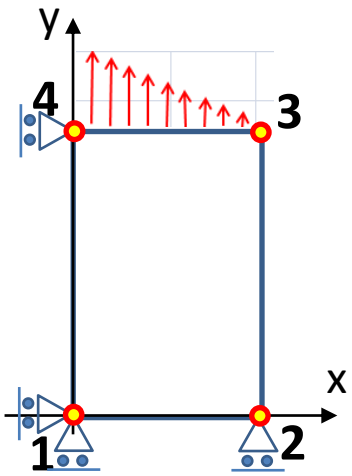
Metoda elementów skończonych (MES1)

Wykład 6C. Tarcza 2D modelowana elementami 4-węzłowymi

03.2024

Przykład. 2D tarcza model MES z użyciem elementów 4-węzłowych





Model ćwiartki tarczy
o jednym elemencie skończonym

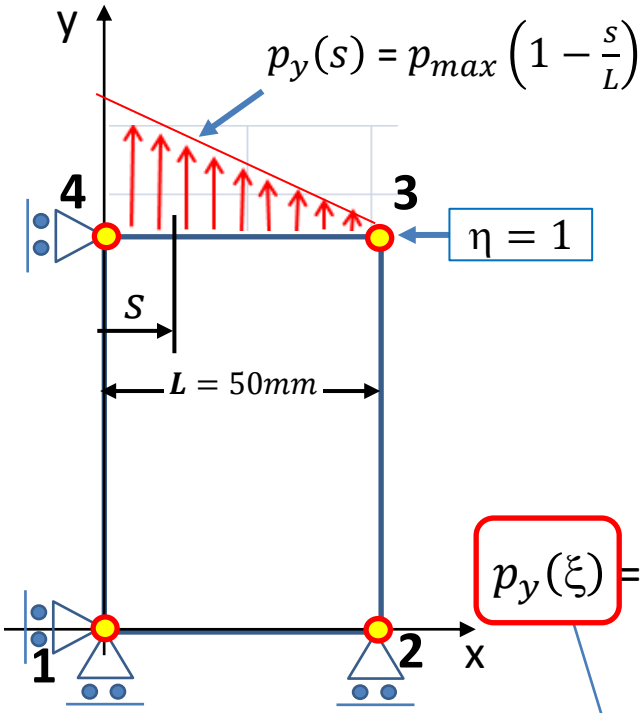
wektory współrzędnych węzłowych:

$$\lfloor x_i \rfloor_1 = \lfloor 0, L, L, 0 \rfloor$$

$$\lfloor y_i \rfloor_1 = \lfloor 0, 0, H, H \rfloor$$

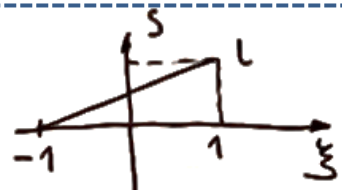
Wektor obciążenia równoważnego od sił powierzchniowych

$$\underset{1\times 8}{\lfloor F^p \rfloor_e} = t_e \int_{-1}^{1} \underset{2\times 8}{\lfloor p_x, p_y \rfloor} \underset{2\times 8}{[N]} \sqrt{\left(\underset{1\times 4}{\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}} \underset{4\times 1}{\{x_i\}_e}\right)^2 + \left(\underset{1\times 4}{\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}} \underset{4\times 1}{\{y_i\}_e}\right)^2}\, d\xi$$



$$x_3 = L\ ,\ x_4 = 0$$
$$y_3 = H\ ,\ y_4 = H$$

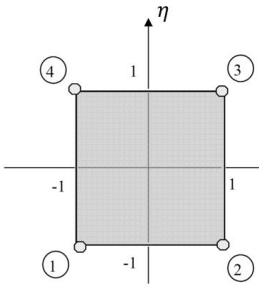
$$\left.\begin{matrix} s{=}0 \ \rightarrow\ \xi = -1 \\ s{=}L \ \rightarrow\ \xi = +1 \end{matrix}\right\} \rightarrow\ s(\xi) = \frac{L}{2}(\xi + 1)$$



$$p_y(\xi) = p_{max}\left(1 - \frac{s(\xi)}{L}\right) = p_{max}\left(1 - \frac{1}{2}(\xi + 1)\right) = \frac{p_{max}}{2}(1 - \xi)$$

$$\underset{1\times 8}{\lfloor F^p \rfloor_1} = t \int_{-1}^{1} \underset{2\times 8}{\lfloor 0, p_y(\xi) \rfloor} \underset{2\times 8}{[N]} \sqrt{\left(\underset{1\times 4}{\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}} \underset{4\times 1}{\{x_i\}_e}\right)^2 + \left(\underset{1\times 4}{\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}} \underset{4\times 1}{\{y_i\}_e}\right)^2}\, d\xi$$

układ współrzędnych naturalnych

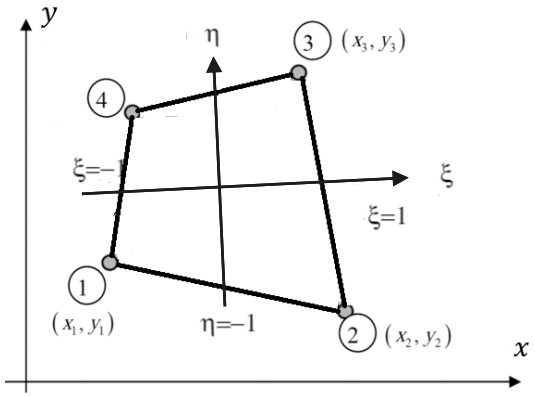


funkcje kształtu elementu i ich pochodne w układzie naturalnym:

$$N_1(\xi,\eta) = \tfrac{1}{4}(1-\xi)(1-\eta)$$
$$N_2(\xi,\eta) = \tfrac{1}{4}(1+\xi)(1-\eta)$$
$$N_3(\xi,\eta) = \tfrac{1}{4}(1+\xi)(1+\eta)$$
$$N_4(\xi,\eta) = \tfrac{1}{4}(1-\xi)(1+\eta)$$

i	$N_i(\xi,1)$
1	0
2	0
3	$\tfrac{1}{2}(1+\xi)$
4	$-\tfrac{1}{2}(1-\xi)$

układ współrzędnych kartezjańskich



i	$\frac{\partial N_i}{\partial \xi}$	$\frac{\partial N_i}{\partial \eta}$
1	$-\tfrac{1}{4}(1-\eta)$	$-\tfrac{1}{4}(1-\xi)$
2	$\tfrac{1}{4}(1-\eta)$	$-\tfrac{1}{4}(1+\xi)$
3	$\tfrac{1}{4}(1+\eta)$	$\tfrac{1}{4}(1+\xi)$
4	$-\tfrac{1}{4}(1+\eta)$	$\tfrac{1}{4}(1-\xi)$

$\frac{\partial N_i(\xi,1)}{\partial \xi}$
$-\tfrac{1}{4}(1-1) = \mathbf{0}$
$\tfrac{1}{4}(1-1) = \mathbf{0}$
$\tfrac{1}{4}(1+1) = \mathbf{\tfrac{1}{2}}$
$-\tfrac{1}{4}(1+1) = \mathbf{-\tfrac{1}{2}}$

$x_3 = L\ ,\ x_4 = 0$
$y_3 = H\ ,\ y_4 = H$

$$\frac{\partial N_3(\xi,1)}{\partial \xi} = ½\ ,\quad \frac{\partial N_4(\xi,1)}{\partial \xi} = -½$$

$$N_3(\xi,1) = ½(1+\xi)\quad ,\quad N_4(\xi,1) = -½(1-\xi)$$

$$\lfloor F^p \rfloor_1 = t\int_{-1}^{1} \lfloor 0, p_y(\xi) \rfloor [N] \sqrt{\left(\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{x_i\}_e\right)^2 + \left(\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{y_i\}_e\right)^2}\, d\xi$$

$$\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{x_i\}_1 = \frac{\partial N_3(\xi,1)}{\partial \xi}x_3 + \frac{\partial N_4(\xi,1)}{\partial \xi}x_4 = ½\cdot L - ½\cdot 0 = \frac{L}{2}$$

$$\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{y_i\}_1 = \frac{\partial N_3(\xi,1)}{\partial \xi}y_3 + \frac{\partial N_4(\xi,1)}{\partial \xi}y_4 = ½\cdot H - ½\cdot \mathrm{H} = 0$$

$$\sqrt{\left(\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{x_i\}_e\right)^2 + \left(\frac{\partial \lfloor N(\xi,1) \rfloor}{\partial \xi}\{y_i\}_e\right)^2} = \frac{L}{2}$$

$$\lfloor F^p \rfloor_1 = t\frac{L}{2}\int_{-1}^{1} \lfloor 0, p_y(\xi) \rfloor [N(\xi,1)]\, d\xi$$

$$[N(\xi,1)] = \begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) \end{bmatrix}$$

$$N_3(\xi,1)= ½(1+\xi) \quad , \quad N_4(\xi,1)= -½(1-\xi)$$

$$[N(\xi,1)] = \begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) \end{bmatrix}$$

$$[F^p]_1 = t\frac{L}{2}\int_{-1}^{1}[0, p_y(\xi)][N(\xi,1)]\, d\xi$$

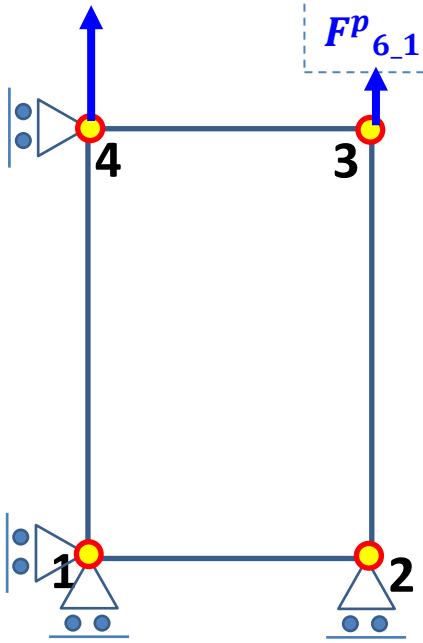
$$p_y(\xi) = \frac{p_{max}}{2}(1-\xi)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) & \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi,1) & 0 & N_4(\xi,1) \end{bmatrix}$$

$[0, p_y(\xi)]$ $\quad [0,0,0,0,0, N_3(\xi,1)p_y(\xi), 0, N_4(\xi,1)p_y(\xi)]$

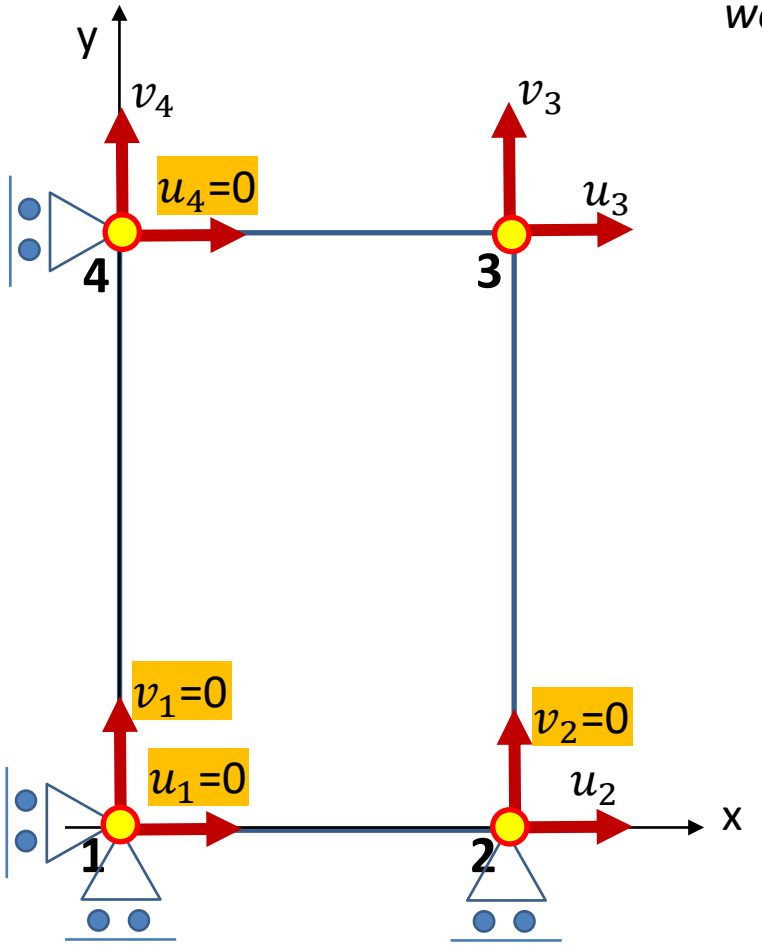
$F^p{}_{8\_1} = \frac{tL}{3}p_{max}$ $\qquad$ $F^p{}_{6\_1}$ $\qquad$ $F^p{}_{8\_1}$

$F^p{}_{6\_1} = \frac{tL}{6}p_{max}$



$$F^p{}_{6\_1} = t\frac{L}{2}\int_{-1}^{1}\frac{p_{max}}{2}(1-\xi)\,½(1+\xi)\,d\xi = \frac{tL}{6}p_{max}$$

$$F^p{}_{8\_1} = t\frac{L}{2}\int_{-1}^{1}\frac{p_{max}}{2}(1-\xi)\big(-½(1-\xi)\big)\,d\xi = \frac{tL}{3}p_{max}$$



wektor parametrów węzłowych elementu 1

$$\underset{8\times 1}{\{q\}_1} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}_1 \qquad wektor\ -kolumna$$

$$\underset{1\times 8}{\lfloor q \rfloor_1} = \lfloor u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4 \rfloor_1$$

$wektor\ -wiersz$

Warunki brzegowe: $\mathrm{N} = 8 - 4 = 4$

wektor aktywnych parametrów węzłowych po uwzględnieniu warunków brzegowych:

$$\underset{1\times 4}{\lfloor q \rfloor_1} = \lfloor u_2, u_3, v_3, v_4 \rfloor_1$$

Macierz gradientu dla warunku PSN:

$$\underset{3\times 2}{[R]} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\frac{\partial}{\partial \xi} - \frac{1}{det[J]}\frac{\partial y}{\partial \xi}\frac{\partial}{\partial \eta}\right) & 0 \\ 0 & \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta} - \frac{1}{det[J]}\frac{\partial x}{\partial \eta}\frac{\partial}{\partial \xi}\right) \\ \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta} - \frac{1}{det[J]}\frac{\partial x}{\partial \eta}\frac{\partial}{\partial \xi}\right) & \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\frac{\partial}{\partial \xi} - \frac{1}{det[J]}\frac{\partial y}{\partial \xi}\frac{\partial}{\partial \eta}\right) \end{bmatrix} =$$

$$= \begin{bmatrix} \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\frac{\partial}{\partial \xi}\right) & 0 \\ 0 & \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\frac{\partial}{\partial \xi}\right) \\ \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\frac{\partial}{\partial \xi}\right) & \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\frac{\partial}{\partial \xi}\right) \end{bmatrix} + \begin{bmatrix} \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\frac{\partial}{\partial \eta}\right) & 0 \\ 0 & \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta}\right) \\ \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta}\right) & \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\frac{\partial}{\partial \eta}\right) \end{bmatrix} =$$

$$= \begin{bmatrix} \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\right) & 0 \\ 0 & \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\right) \\ \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\right) & \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\right) \end{bmatrix}\frac{\partial}{\partial \xi} + \begin{bmatrix} \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\right) & 0 \\ 0 & \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta}\right) \\ \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\right) & \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\right) \end{bmatrix}\frac{\partial}{\partial \eta}$$

macierz odkształcenie–przemieszczenie

$$\underset{3\times 16}{[B]} = \underset{3\times 2}{[R]}\underset{2\times 16}{[N]} = \begin{bmatrix} \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\right) & 0 \\ 0 & \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\right) \\ \left(-\frac{1}{det[J]}\frac{\partial x}{\partial \eta}\right) & \left(\frac{1}{det[J]}\frac{\partial y}{\partial \eta}\right) \end{bmatrix}\left[\frac{\partial N}{\partial \xi}\right] + \begin{bmatrix} \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\right) & 0 \\ 0 & \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\frac{\partial}{\partial \eta}\right) \\ \left(\frac{1}{det[J]}\frac{\partial x}{\partial \xi}\right) & \left(-\frac{1}{det[J]}\frac{\partial y}{\partial \xi}\right) \end{bmatrix}\left[\frac{\partial N}{\partial \eta}\right]$$

$$det[J] = det\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi}\frac{\partial x}{\partial \eta} =$$

$$= \frac{\partial(\lfloor N(\xi,\eta)\rfloor\{x_i\}_e)}{\partial \xi}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{y_i\}_e)}{\partial \eta} - \frac{\partial(\lfloor N(\xi,\eta)\rfloor\{y_i\}_e)}{\partial \xi}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{x_i\}_e)}{\partial \eta} =$$

$$= \underset{1\times 4}{\left(\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \xi}\right)}\underset{4\times 1}{\{x_i\}_e}\underset{1\times 4}{\left(\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \eta}\right)}\underset{4\times 1}{\{y_i\}_e} - \underset{1\times 4}{\left(\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \xi}\right)}\underset{4\times 1}{\{y_i\}_e}\underset{1\times 4}{\left(\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \eta}\right)}\underset{4\times 1}{\{x_i\}_e}$$

$$\frac{\partial \xi}{\partial x} = \frac{1}{det[J]}\frac{\partial y}{\partial \eta} = \frac{1}{det[J]}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{y_i\}_e)}{\partial \eta} = \frac{1}{det[J]}\underset{1\times 4}{\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \eta}}\underset{4\times 1}{\{y_i\}_e}$$

$$\frac{\partial \eta}{\partial x} = -\frac{1}{det[J]}\frac{\partial y}{\partial \xi} = -\frac{1}{det[J]}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{y_i\}_e)}{\partial \xi} = -\frac{1}{det[J]}\underset{1\times 4}{\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \xi}}\underset{4\times 1}{\{y_i\}_e}$$

$$\frac{\partial \xi}{\partial y} = -\frac{1}{det[J]}\frac{\partial x}{\partial \eta} = -\frac{1}{det[J]}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{x_i\}_e)}{\partial \eta} = -\frac{1}{det[J]}\underset{1\times 4}{\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \eta}}\underset{4\times 1}{\{x_i\}_e}$$

$$\frac{\partial \eta}{\partial y} = \frac{1}{det[J]}\frac{\partial x}{\partial \xi} = \frac{1}{det[J]}\frac{\partial(\lfloor N(\xi,\eta)\rfloor\{x_i\}_e)}{\partial \xi} = \frac{1}{det[J]}\underset{1\times 4}{\frac{\partial\lfloor N(\xi,\eta)\rfloor}{\partial \xi}}\underset{4\times 1}{\{x_i\}_e}$$

$$\underset{2\times 8}{\left[\frac{\partial N}{\partial \xi}\right]} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\ 0 & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} \end{bmatrix}$$

$$\underset{2\times 8}{\left[\frac{\partial N}{\partial \eta}\right]} = \begin{bmatrix} \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 \\ 0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} \end{bmatrix}$$

$$\underset{3\times 3}{[D]} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix}$$

macierz konstytutywna dla PSN:

macierz sztywności elementu:

$$\underset{8\times 8}{[k]_e} = t_e \int_{-1}^{1}\int_{-1}^{1} \underset{8\times 3}{[B(\xi,\eta)]^T} \underset{3\times 3}{[D]} \underset{3\times 8}{[B(\xi,\eta)]} det[J]\, d\xi d\eta$$

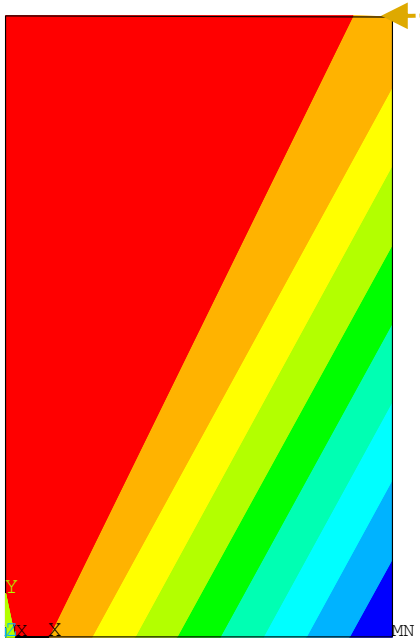
Całkowane numerycznie mieszaną regułą kwadratur aby uniknąć „shear locking”
(Enhanced strain formulation)

$$\{q\} = [k]^{-1}\{F\}$$

$4 \times 1 \qquad 4 \times 4 \qquad 4 \times 1$

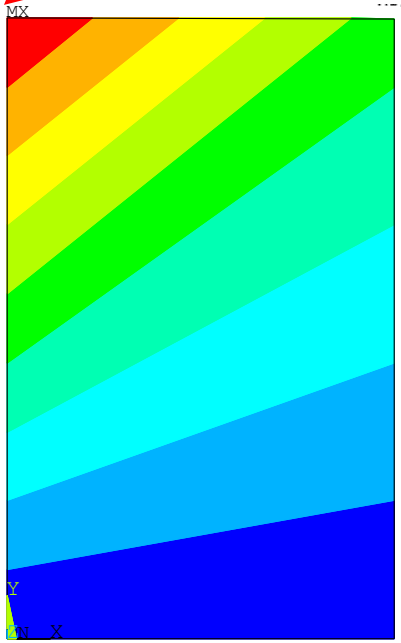
F		q	
0		-.012724	u2
0		-.001562	u3
1000	N	.02286	v3
2000	N	.045711	v4

UX displacement



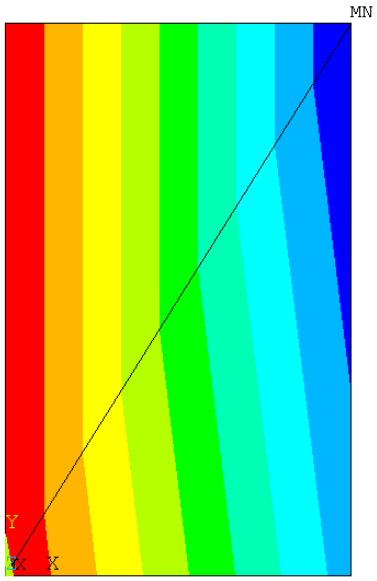
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMN =-.012724
-.012724
-.01131
-.009896
-.008483
-.007069
-.005655
-.004241
-.002828
-.001414
0

UY displacement

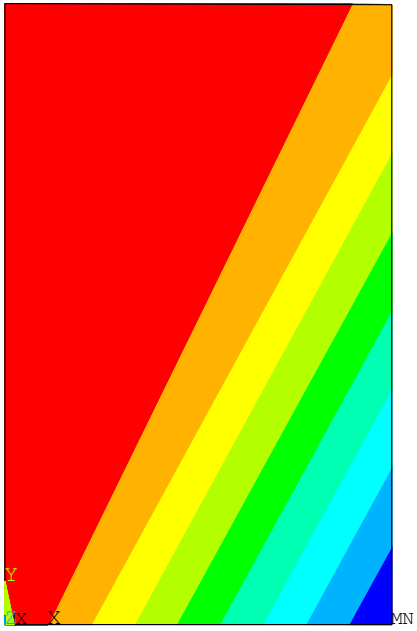


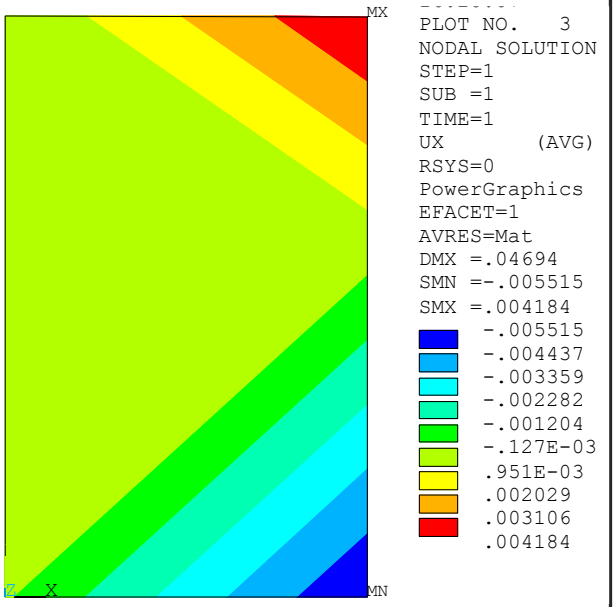
PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMX =.045711
0
.005079
.010158
.015237
.020316
.025395
.030474
.035553
.040632
.045711

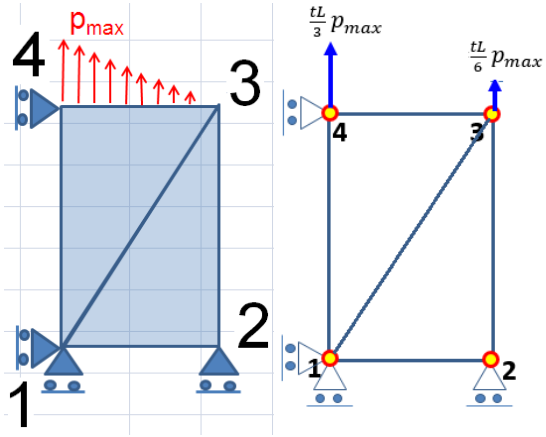
Przemieszczenia na kierunek X

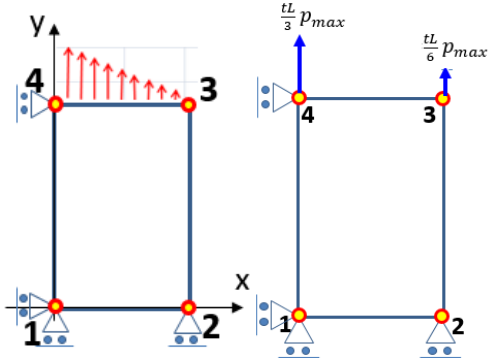


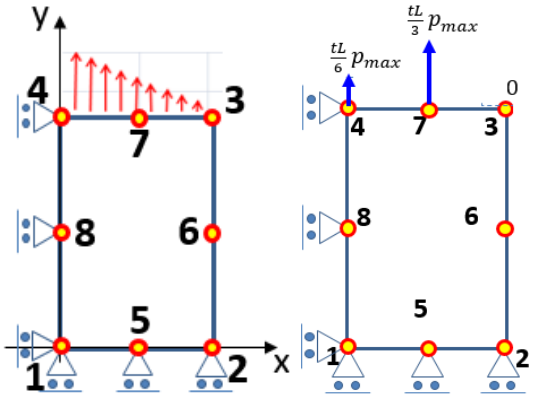
UX displacement



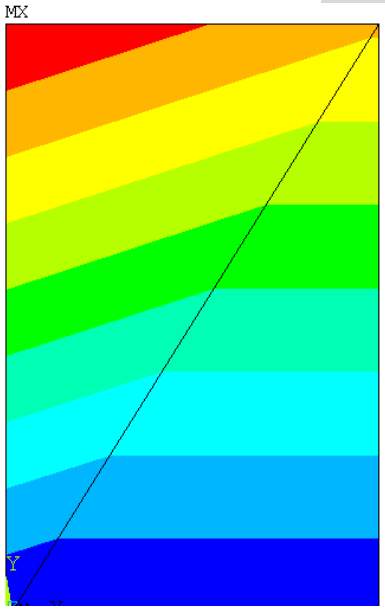








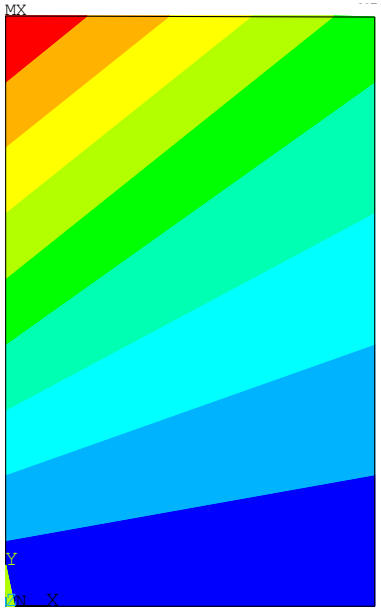
Przemieszczenia na kierunek Y



PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMX =.038165

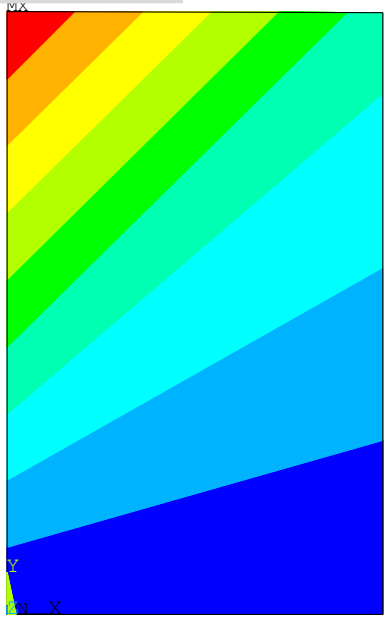
0
.004241
.008481
.012722
.016962
.021203
.025444
.029684
.033925
.038165

UY displacement



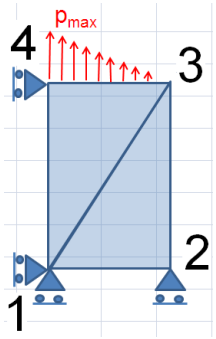
PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMX =.045711

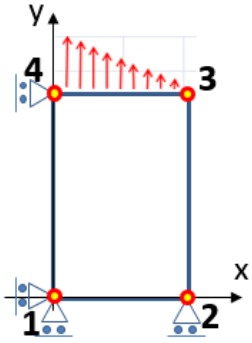
0
.005079
.010158
.015237
.020316
.025395
.030474
.035553
.040632
.045711

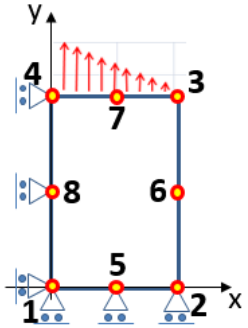


PLOT NO. 4
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.04694
SMX =.04694

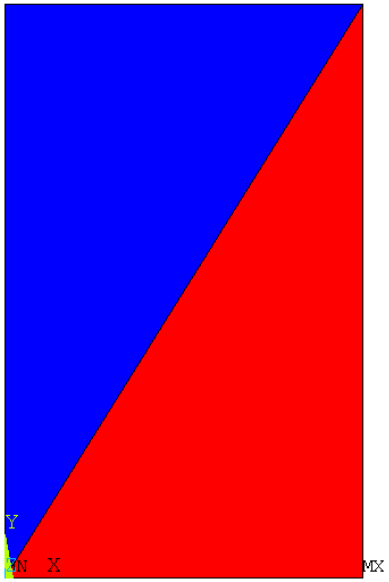
0
.005216
.010431
.015647
.020862
.026078
.031293
.036509
.041724
.04694





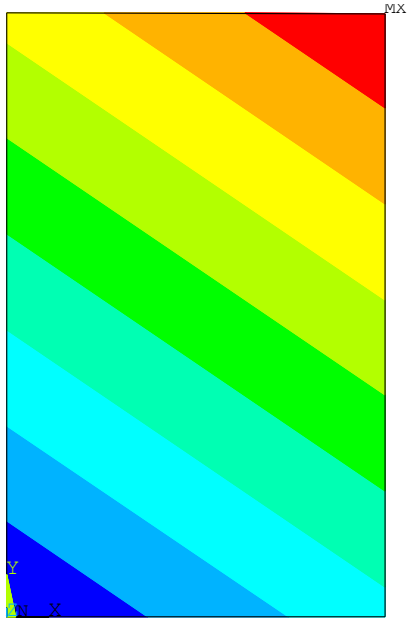


Odkształcenia na kierunek X

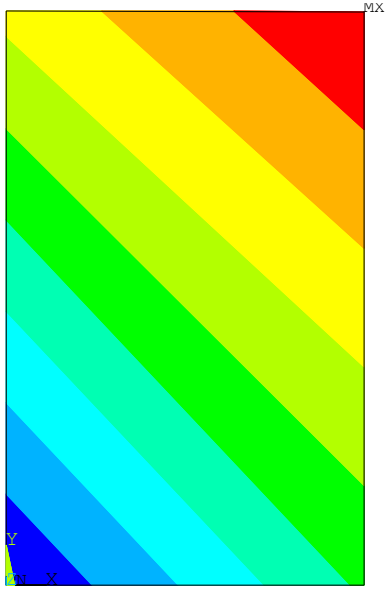


PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.156E-03
SMX =-.130E-03
-.156E-03
-.153E-03
-.150E-03
-.147E-03
-.144E-03
-.141E-03
-.139E-03
-.136E-03
-.133E-03
-.130E-03

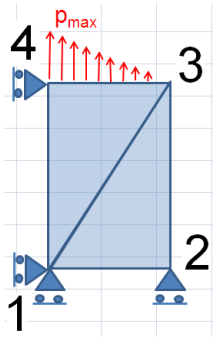
ε_x **strain**

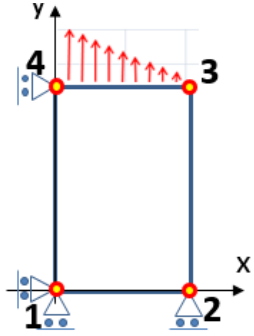


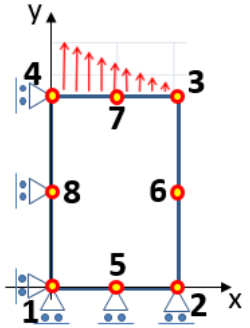
PLOT NO. 3
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMN =-.302E-03
SMX =.164E-04
-.302E-03
-.267E-03
-.231E-03
-.196E-03
-.161E-03
-.125E-03
-.898E-04
-.544E-04
-.190E-04
.164E-04



PLOT NO. 12
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-.313E-03
SMX =.157E-05
-.313E-03
-.278E-03
-.243E-03
-.208E-03
-.173E-03
-.138E-03
-.103E-03
-.682E-04
-.333E-04
.157E-05

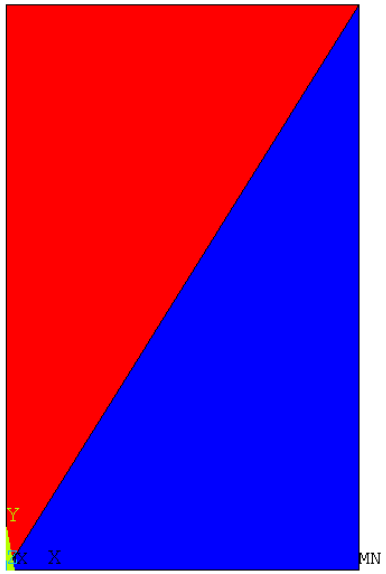






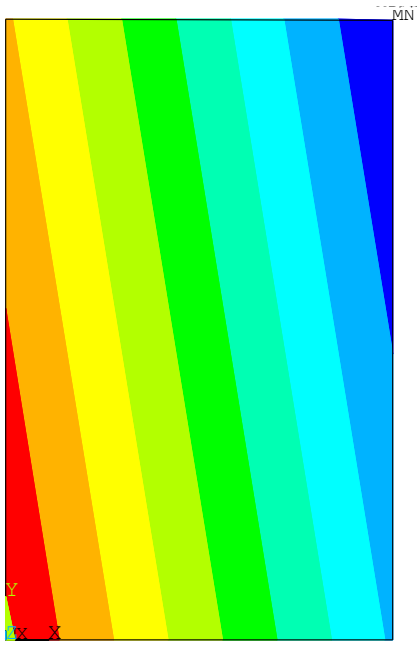
Odkształcenia na kierunek Y

ε_y **strain**



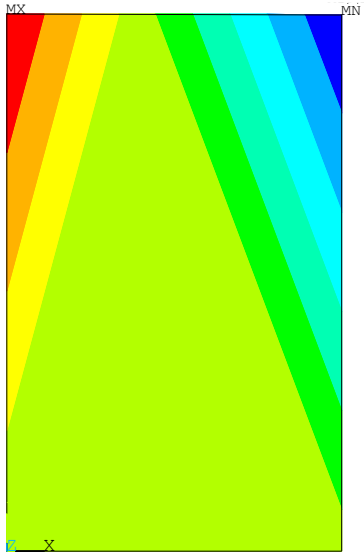
PLOT NO. 8
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =.380E-03
SMX =.477E-03

.380E-03
.391E-03
.402E-03
.412E-03
.423E-03
.434E-03
.445E-03
.456E-03
.466E-03
.477E-03



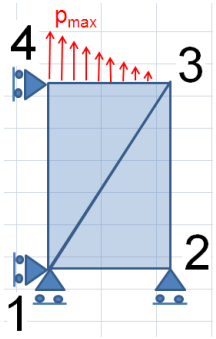
PLOT NO. 4
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMN =.249E-03
SMX =.609E-03

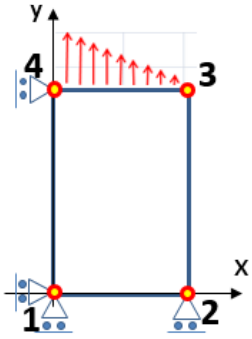
.249E-03
.289E-03
.329E-03
.369E-03
.409E-03
.449E-03
.489E-03
.529E-03
.569E-03
.609E-03

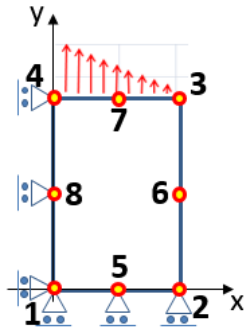


PLOT NO. 13
ELEMENT SOLUTIO
STEP=1
SUB =1
TIME=1
EPELY (NOAVG
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =.238E-04
SMX =.769E-03

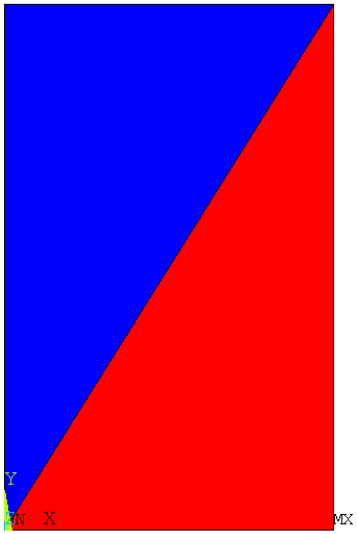
.238E-04
.107E-03
.189E-03
.272E-03
.355E-03
.438E-03
.520E-03
.603E-03
.686E-03
.769E-03







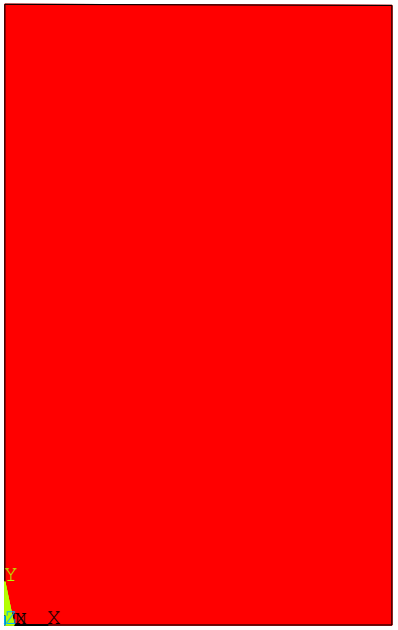
Odkształcenia postaciowe



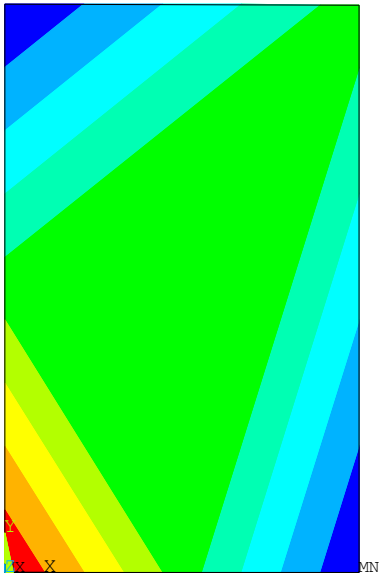
PLOT NO. 1
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.155E-03
SMX =-.160E-04

-.155E-03
-.140E-03
-.124E-03
-.109E-03
-.933E-04
-.779E-04
-.624E-04
-.470E-04
-.315E-04
-.160E-04

γ_{xy} strain

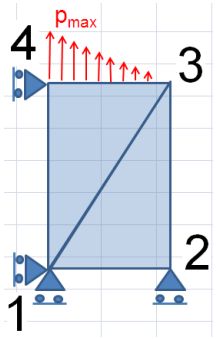


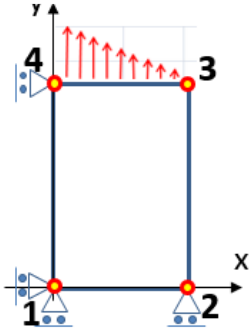
PLOT NO. 5
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMN =-.159E-03
SMX =-.159E-03

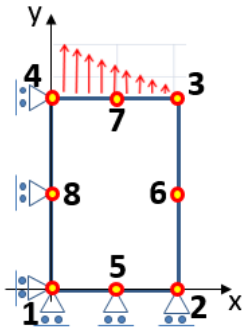


PLOT NO. 14
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-.259E-03
SMX =.105E-03

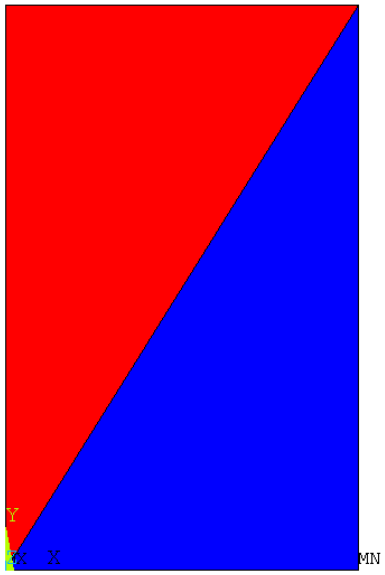
-.259E-03
-.219E-03
-.178E-03
-.138E-03
-.974E-04
-.569E-04
-.164E-04
.241E-04
.646E-04
.105E-03





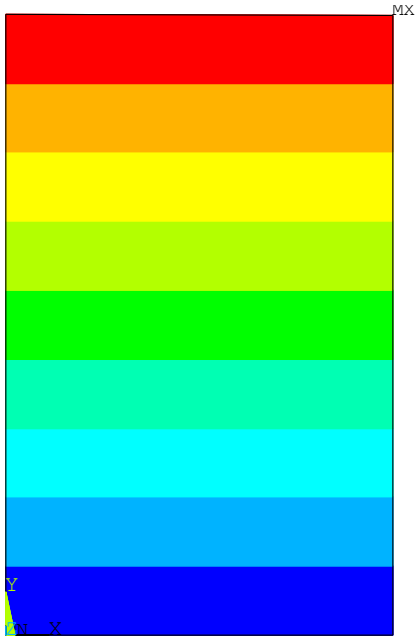


Naprężenia na kierunek X

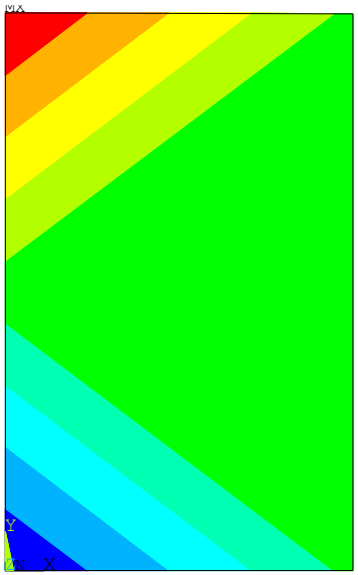


PLOT NO. 3
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.263011
SMX =.263011
-.263011
-.204564
-.146117
-.08767
-.029223
.029223
.08767
.146117
.204564
.263011

σ_x stress



PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-7.81349
SMX =7.81349
-7.81349
-6.07716
-4.34083
-2.6045
-.868165
.868165
2.6045
4.34083
6.07716
7.81349



PLOT NO. 5
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-12.8298
SMX =12.8298
-12.8298
-9.97872
-7.12766
-4.2766
-1.42553
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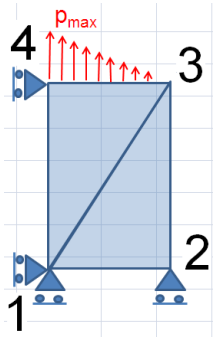
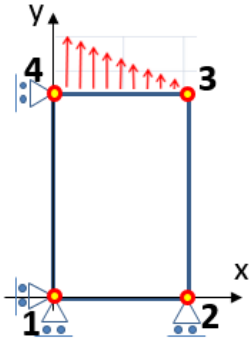
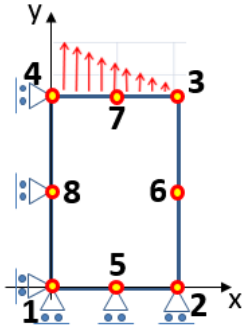
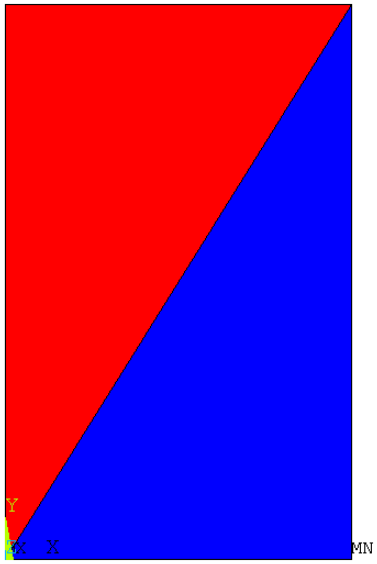
4.2766
7.12766
9.97872
12.8298







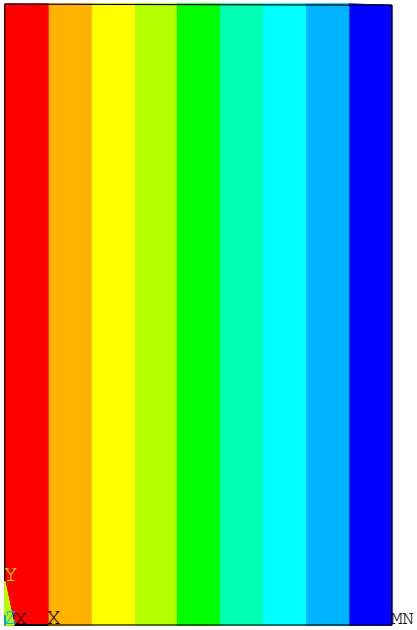
Naprężenia na kierunek Y



PLOT NO. 4
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =26.518
SMX =33.482

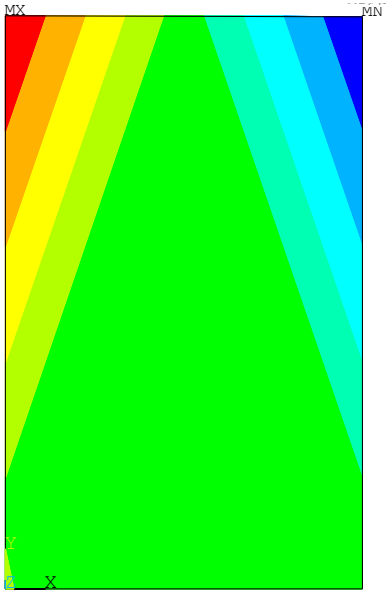
26.518
27.292
28.065
28.839
29.613
30.387
31.161
31.935
32.708
33.482

σ_y stress



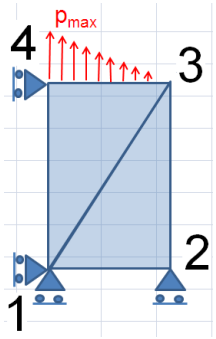
PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =20.0025
SMX =39.9975

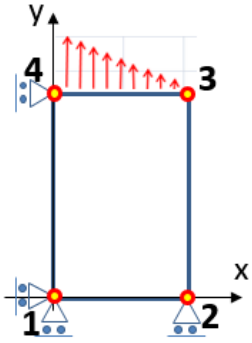
20.0025
22.2242
24.4459
26.6675
28.8892
31.1108
33.3325
35.5541
37.7758
39.9975

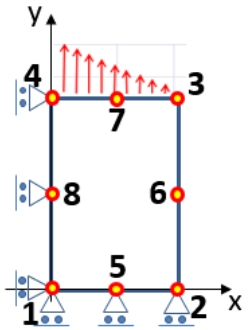


PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =1.91405
SMX =58.086

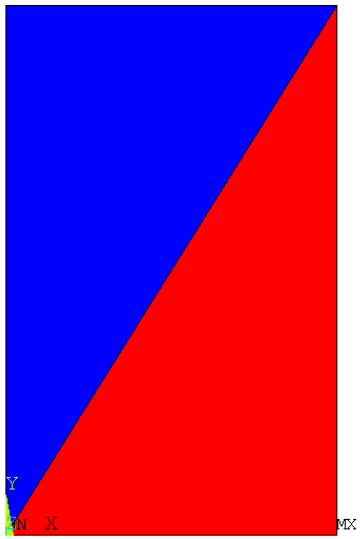
1.91405
8.15537
14.3967
20.638
26.8793
33.1207
39.362
45.6033
51.8446
58.086



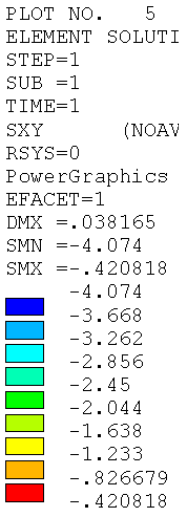




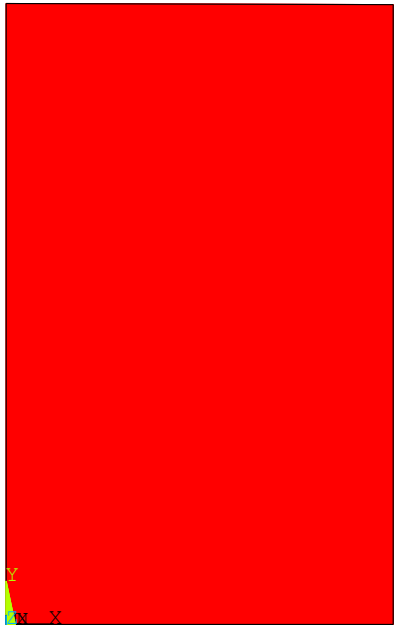
Naprężenia styczne



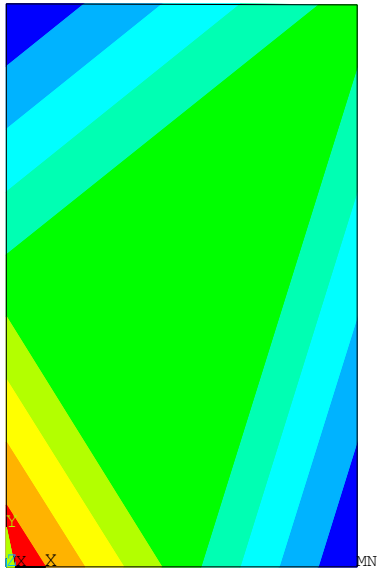
PLOT NO. 5
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-4.074
SMX =-.420818



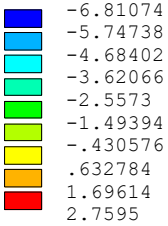
τ_{xy} **stress**

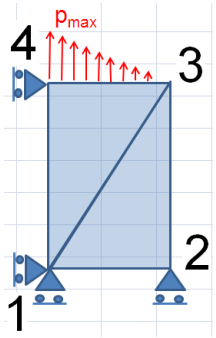


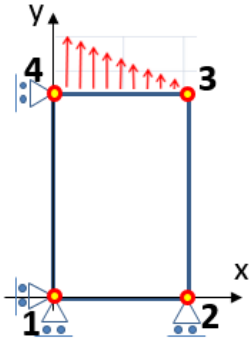
PLOT NO. 8
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-4.16719
SMX =-4.16719

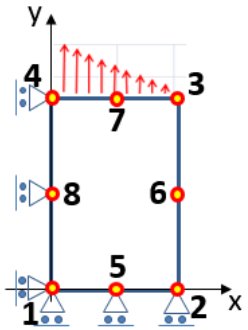


PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-6.81074
SMX =2.7595

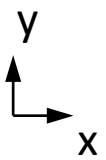


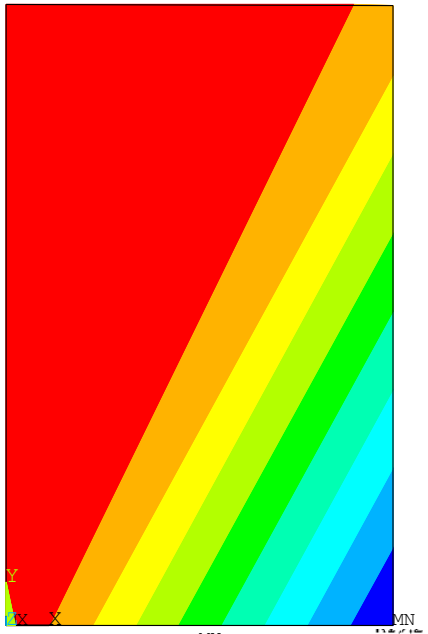






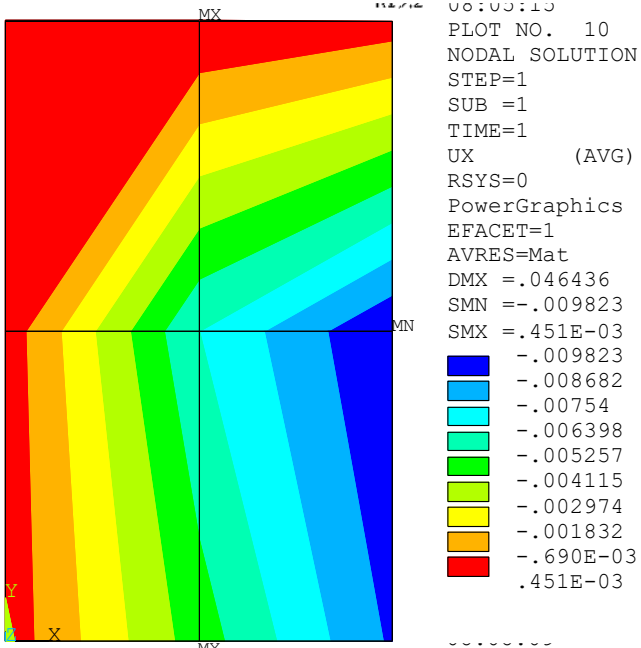
Wpływ dyskretyzacji na jakość wyników

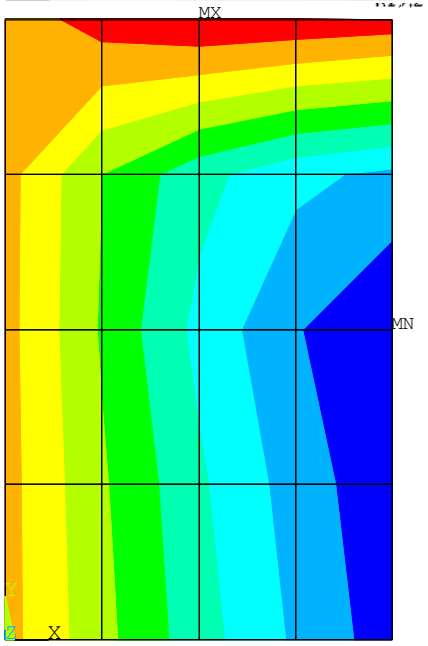


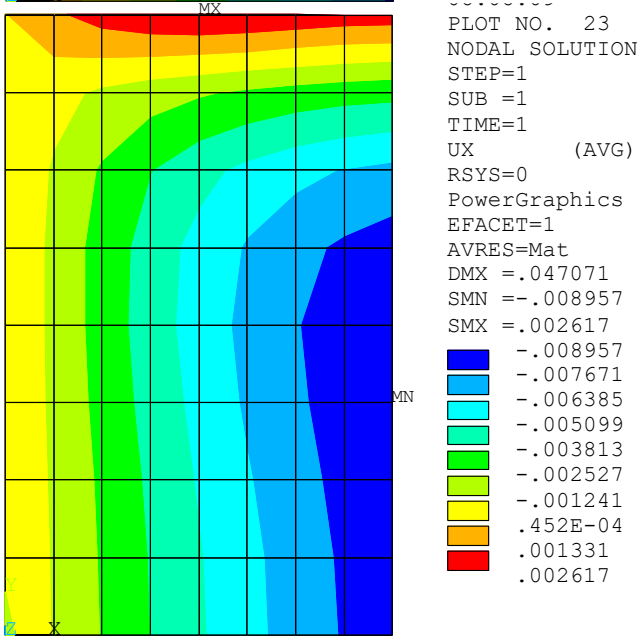


UX

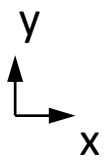
[mm]

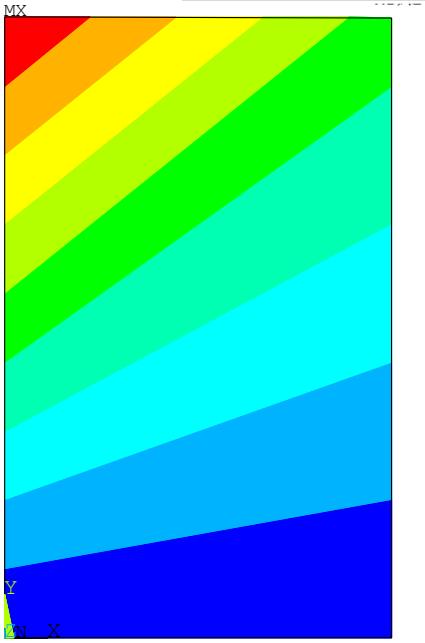




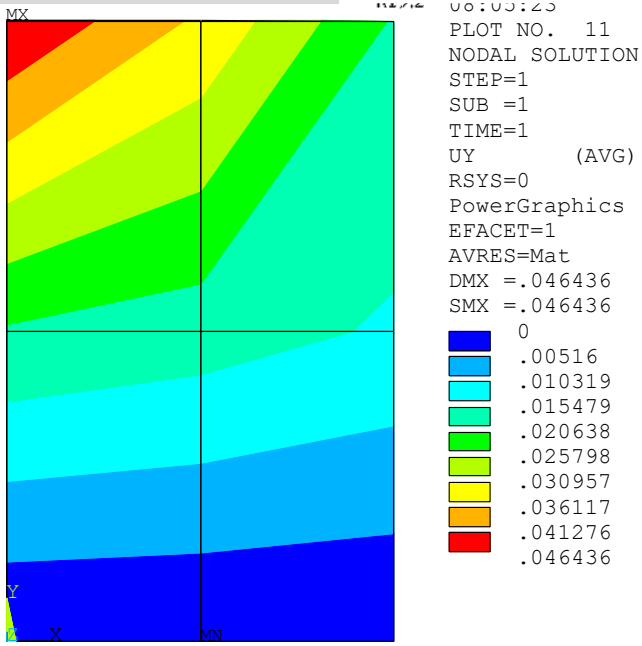


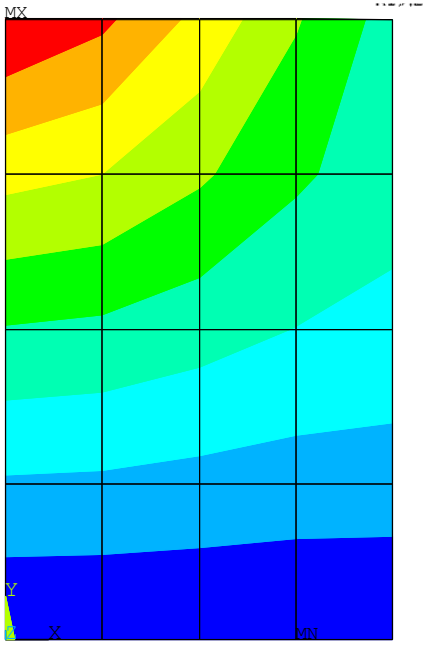
Wpływ dyskretyzacji na jakość wyników

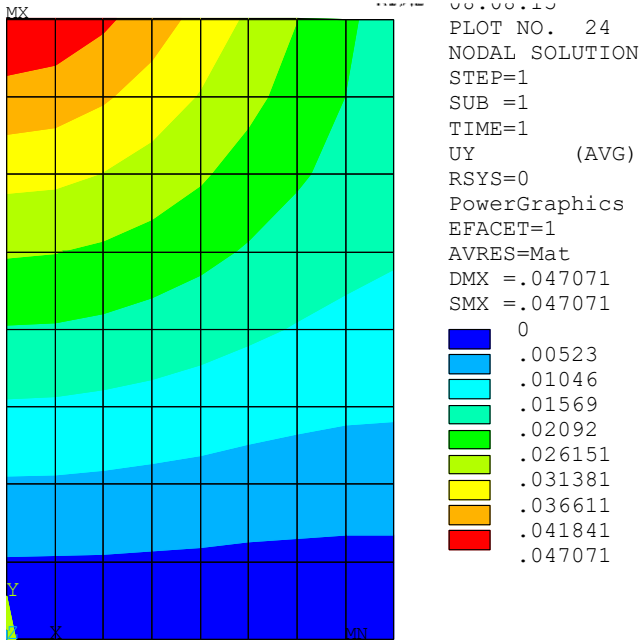




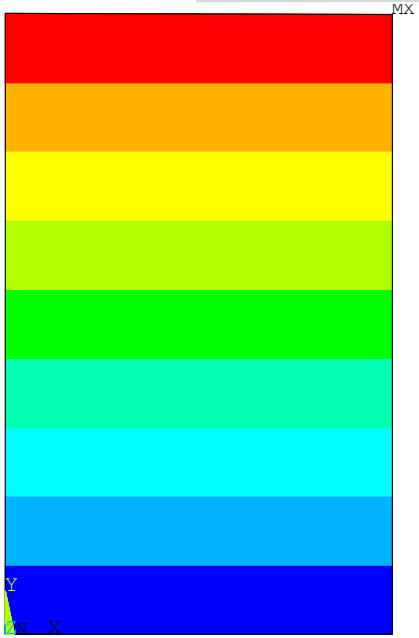
UY [mm]







Wpływ dyskretyzacji na jakość wyników



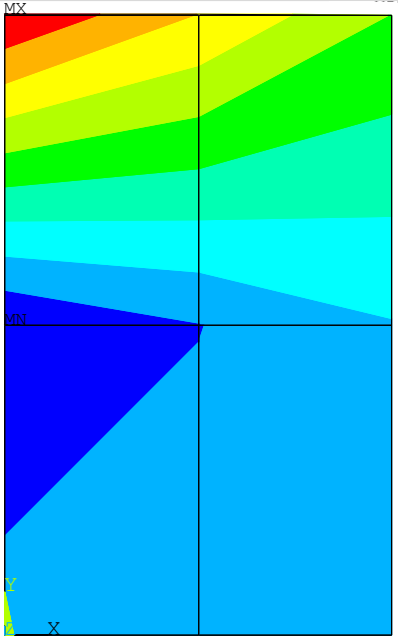
PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-7.81349
SMX =7.81349

-7.81349
-6.07716
-4.34083
-2.6045
-.868165
.868165
2.6045
4.34083
6.07716
7.81349

σ_x

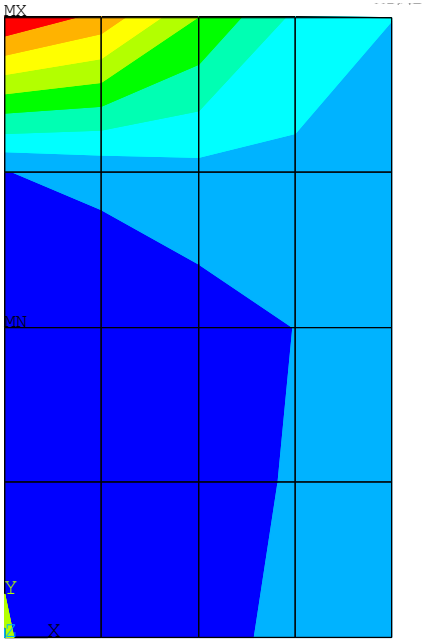
[MPa]

NODAL SOLUTION



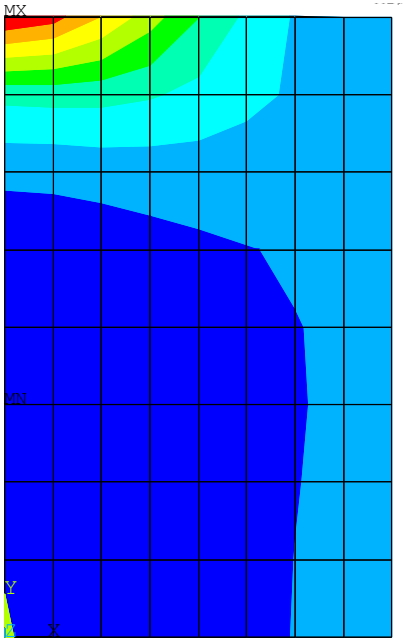
PLOT NO. 31
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046436
SMN =-5.95909
SMX =14.5388

-5.95909
-3.68154
-1.404
.873548
3.15109
5.42864
7.70619
9.98373
12.2613
14.5388



PLOT NO. 20
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046861
SMN =-4.63339
SMX =24.1699

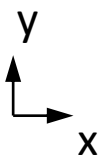
-4.63339
-1.43302
1.76735
4.96772
8.16809
11.3685
14.5688
17.7692
20.9696
24.1699

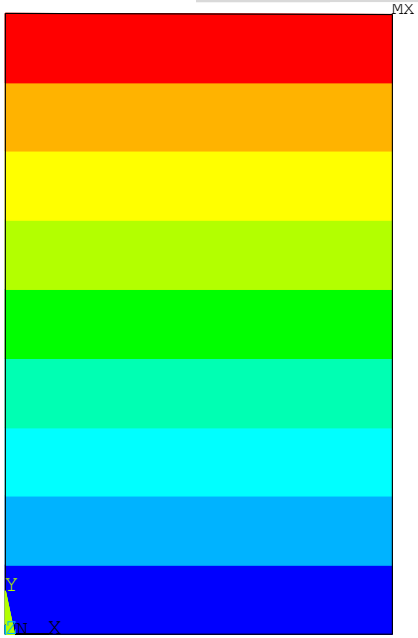


PLOT NO. 25
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047071
SMN =-4.69728
SMX =30.1917

-4.69728
-.820734
3.05582
6.93237
10.8089
14.6855
18.562
22.4386
26.3151
30.1917

Wpływ dyskretyzacji na jakość wyników





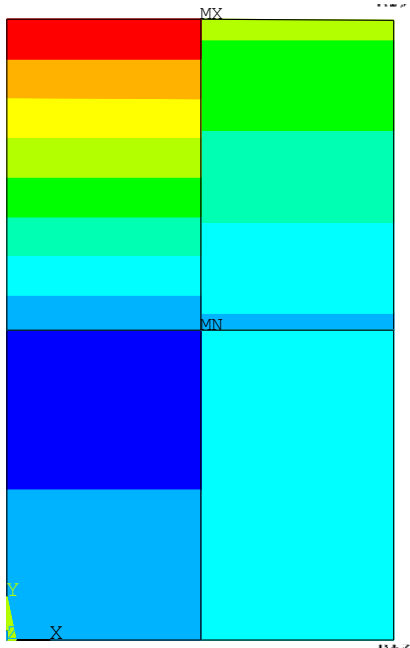
PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-7.81349
SMX =7.81349

-7.81349
-6.07716
-4.34083
-2.6045
-.868165
.868165
2.6045
4.34083
6.07716
7.81349

σ_x

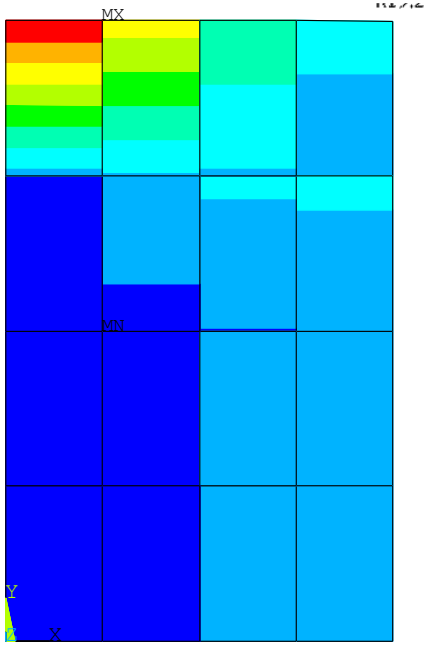
[MPa]

ELEMENT SOLUTION



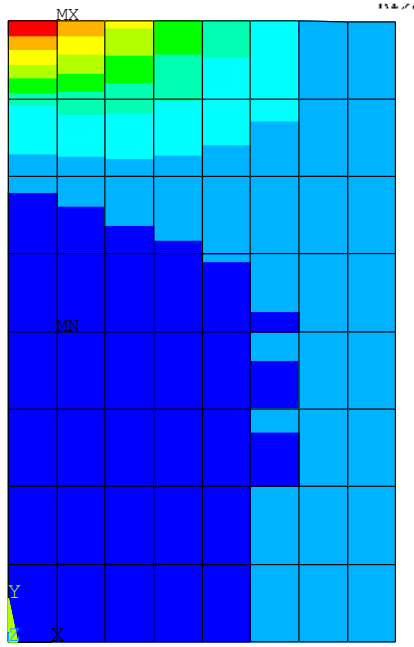
PLOT NO. 12
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046436
SMN =-7.31649
SMX =14.5388

-7.31649
-4.88812
-2.45975
-.031386
2.39698
4.82535
7.25372
9.68209
12.1105
14.5388



PLOT NO. 17
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046861
SMN =-5.58213
SMX =24.1699

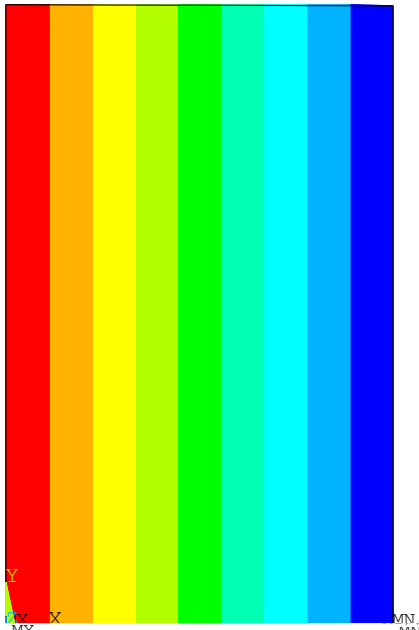
-5.58213
-2.27634
1.02944
4.33523
7.64102
10.9468
14.2526
17.5584
20.8642
24.1699



PLOT NO. 28
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047071
SMN =-5.14419
SMX =30.1917

-5.14419
-1.21798
2.70822
6.63443
10.5606
14.4868
18.413
22.3393
26.2655
30.1917

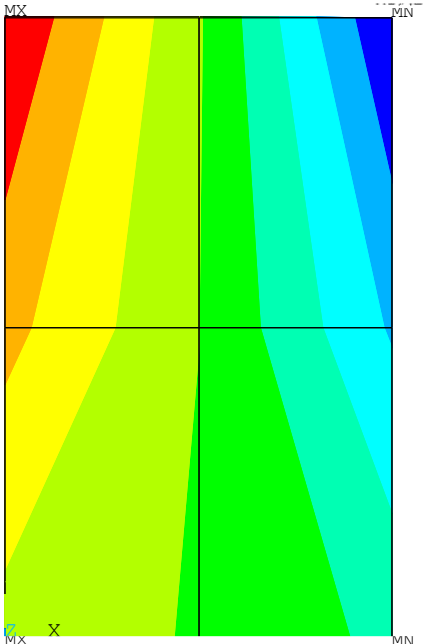
Wpływ dyskretyzacji na jakość wyników

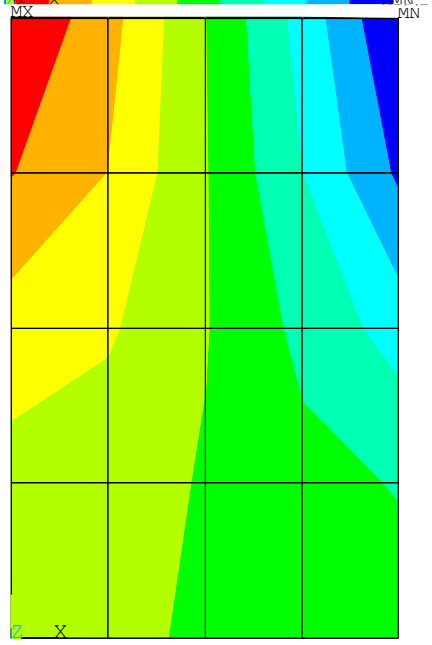


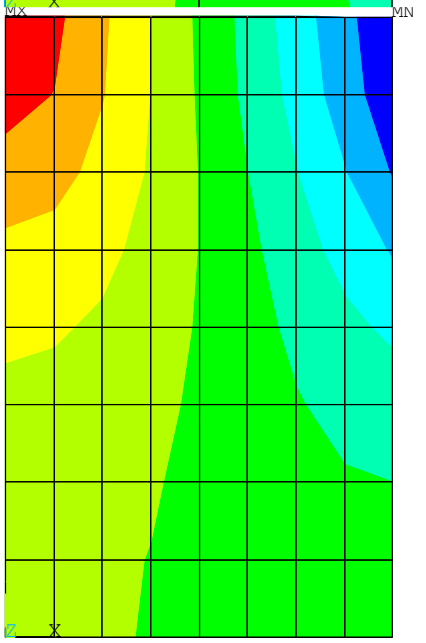
σ_y

[MPa]

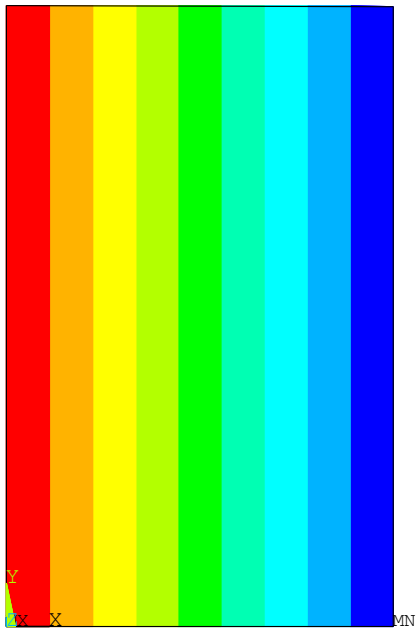
NODAL SOLUTION







Wpływ dyskretyzacji na jakość wyników



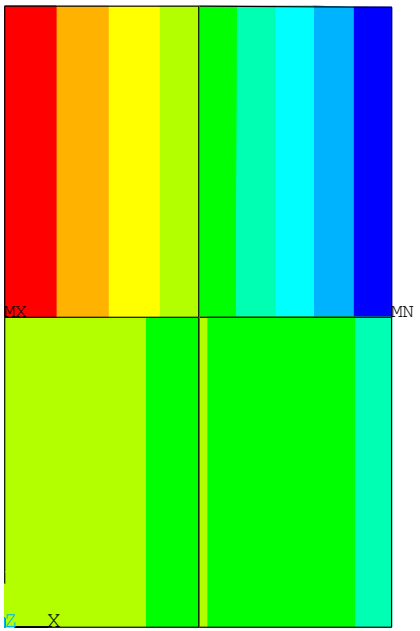
PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =20.0025
SMX =39.9975

20.0025
22.2242
24.4459
26.6675
28.8892
31.1108
33.3325
35.5541
37.7758
39.9975

σ_y

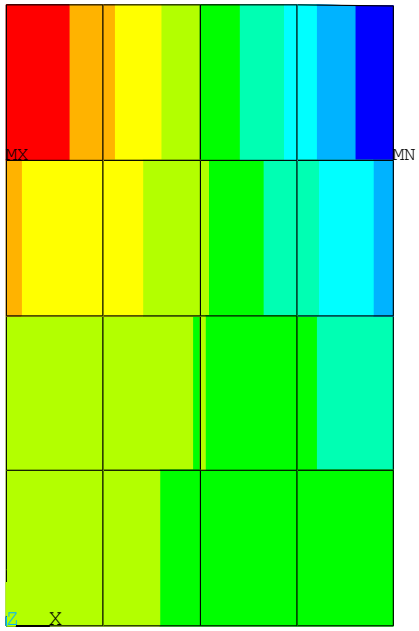
[MPa]

ELEMENT SOLUTION



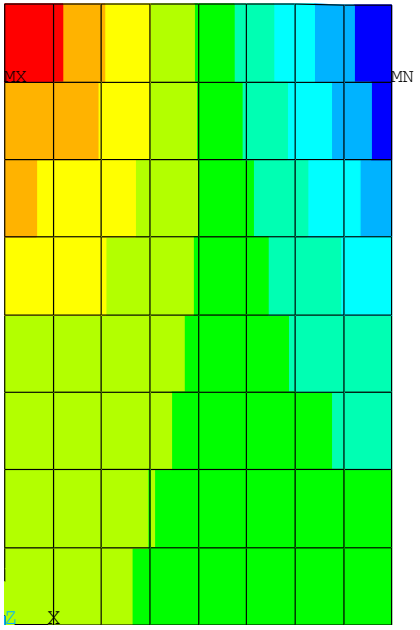
PLOT NO. 13
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046436
SMN =9.68398
SMX =47.7465

9.68398
13.9132
18.1423
22.3715
26.6007
30.8298
35.059
39.2882
43.5173
47.7465



PLOT NO. 18
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046861
SMN =3.04724
SMX =52.8834

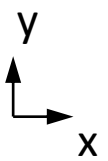
3.04724
8.5846
14.122
19.6593
25.1967
30.734
36.2714
41.8087
47.3461
52.8834

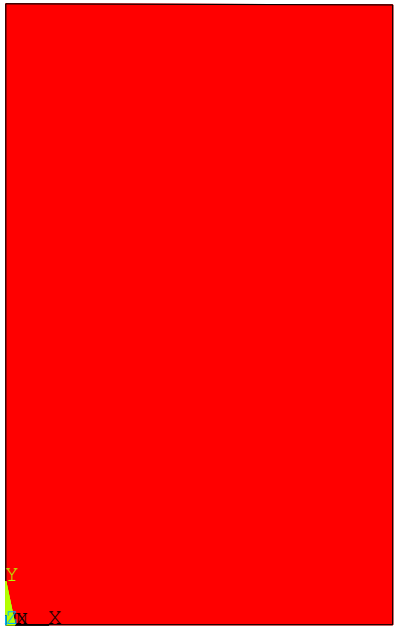


PLOT NO. 29
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047071
SMN =.427497
SMX =56.1359

.427497
6.61732
12.8071
18.997
25.1868
31.3766
37.5664
43.7562
49.946
56.1359

Wpływ dyskretyzacji na jakość wyników



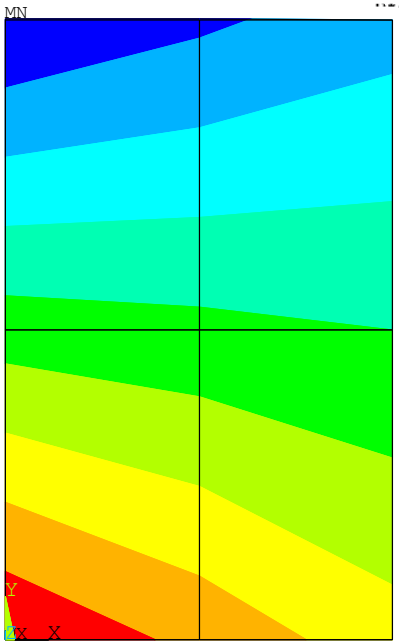


PLOT NO. 8
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-4.16719
SMX =-4.16719

τ_{xy}

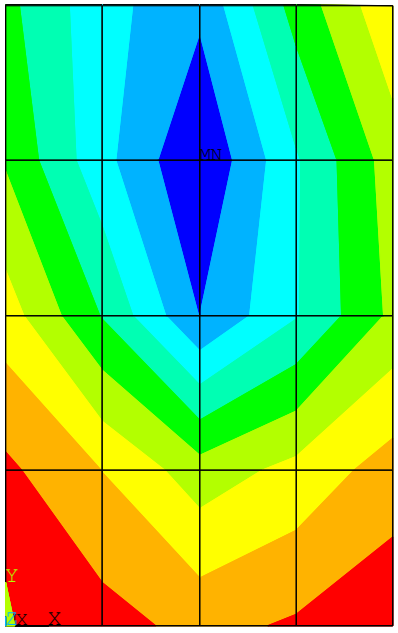
[MPa]

NODAL SOLUTION



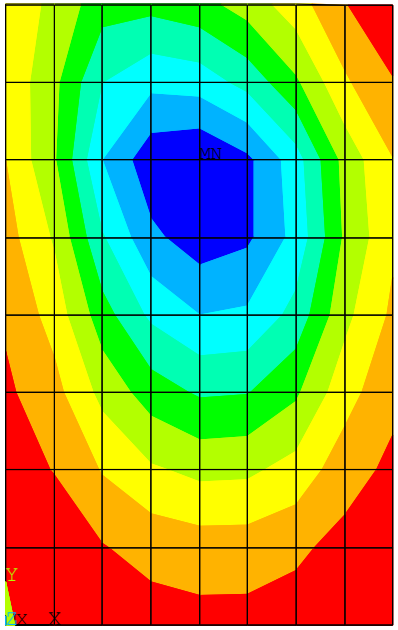
PLOT NO. 33
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046436
SMN =-4.70816
SMX =-1.65009

-4.70816
-4.36837
-4.02859
-3.6888
-3.34902
-3.00923
-2.66944
-2.32966
-1.98987
-1.65009



PLOT NO. 22
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046861
SMN =-5.49877
SMX =-.425279

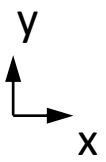
-5.49877
-4.93504
-4.37132
-3.8076
-3.24388
-2.68016
-2.11644
-1.55272
-.989
-.425279

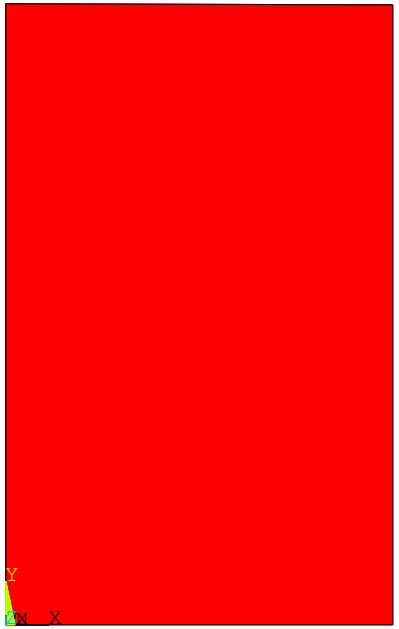


PLOT NO. 27
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047071
SMN =-6.68
SMX =-.106963

-6.68
-5.94966
-5.21933
-4.48899
-3.75865
-3.02831
-2.29798
-1.56764
-.837301
-.106963

Wpływ dyskretyzacji na jakość wyników



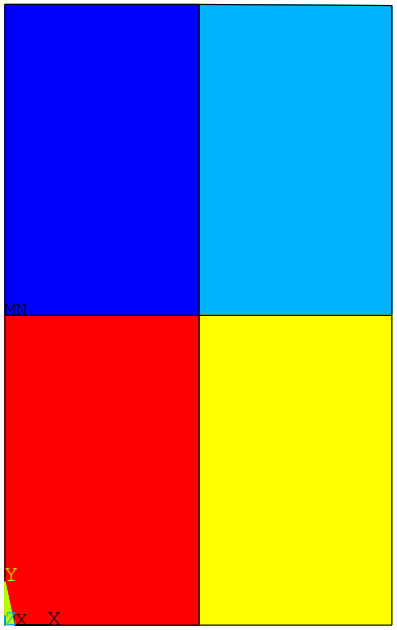


PLOT NO. 8
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-4.16719
SMX =-4.16719

τ_{xy}

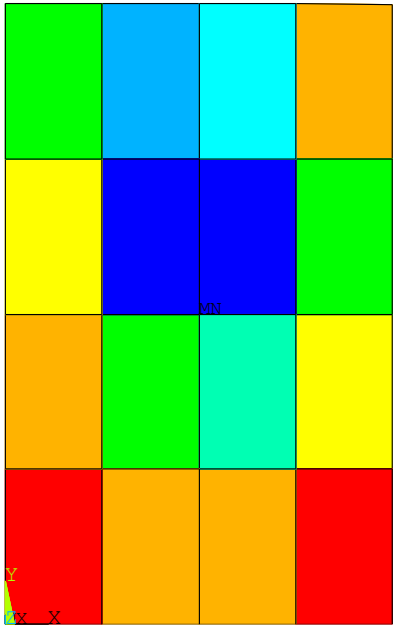
[MPa]

ELEMENT SOLUTION



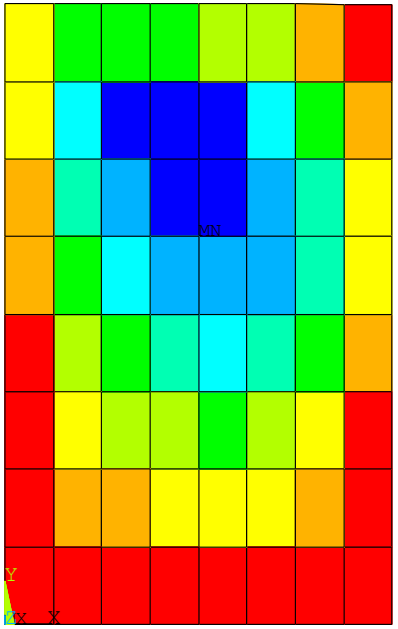
PLOT NO. 14
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046436
SMN =-4.70816
SMX =-1.65009

-4.70816
-4.36837
-4.02859
-3.6888
-3.34902
-3.00923
-2.66944
-2.32966
-1.98987
-1.65009



PLOT NO. 19
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046861
SMN =-6.33246
SMX =-.425279

-6.33246
-5.6761
-5.01975
-4.3634
-3.70704
-3.05069
-2.39434
-1.73799
-1.08163
-.425279



PLOT NO. 30
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047071
SMN =-6.83544
SMX =-.106963

-6.83544
-6.08783
-5.34022
-4.59261
-3.84501
-3.0974
-2.34979
-1.60218
-.854572
-.106963